

Final Examples

Announcements

Trees

Tree-Structured Data

Tree-Structured Data

```
def tree(label, branches=[]):  
    return [label] + list(branches)  
  
def label(tree):  
    return tree[0]  
  
def branches(tree):  
    return tree[1:]  
  
class Tree:  
    def __init__(self, label, branches=[]):  
        self.label = label  
        self.branches = list(branches)
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A tree can contains other trees:

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[5, [6, 7], 8, [[9], 10]]

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<ul>
  <li>Midterm <b>1</b></li>
  <li>Midterm <b>2</b></li>
</ul>
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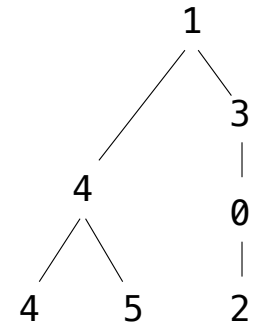
Tree processing often involves recursive calls on subtrees

Tree Processing

Solving Tree Problems

Implement **big**s, which takes a Tree instance t containing integer labels. It returns the number of nodes in t whose labels are larger than any labels of their ancestor nodes.

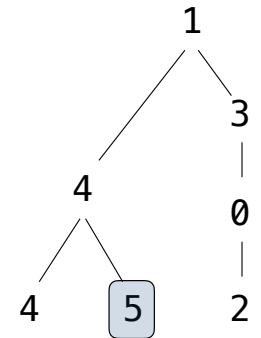
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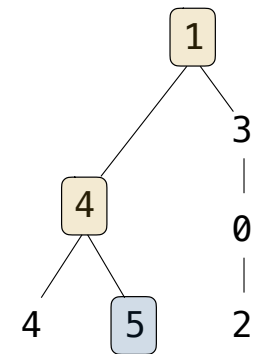
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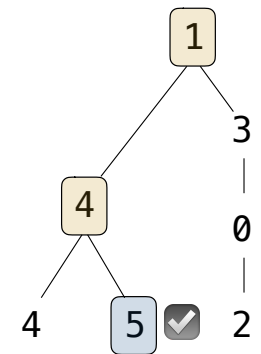
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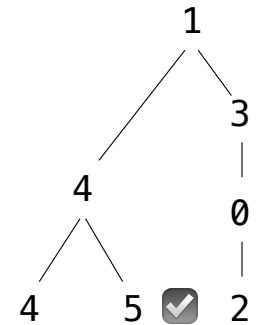
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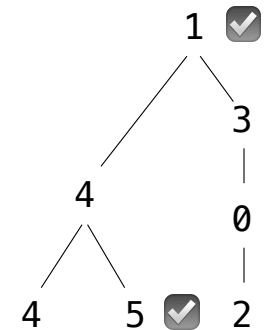
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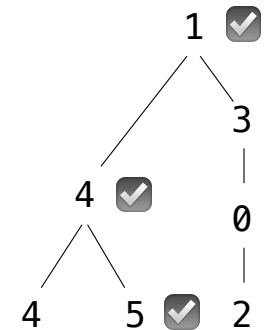
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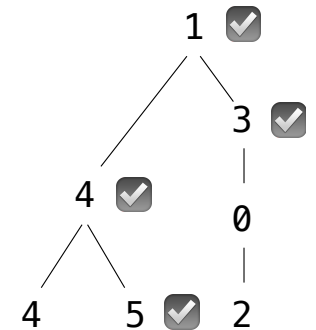
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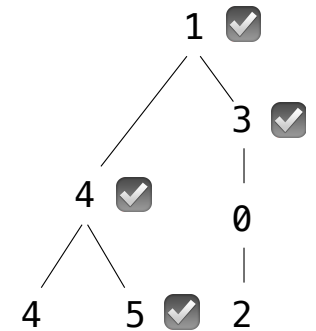


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The root label is always larger than all of its ancestors



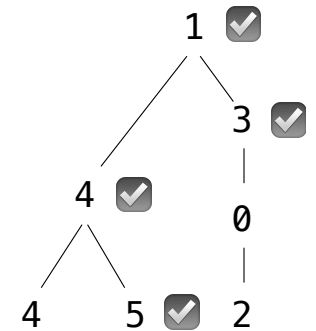
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if t.is_leaf():  
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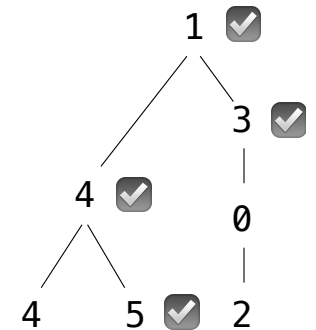
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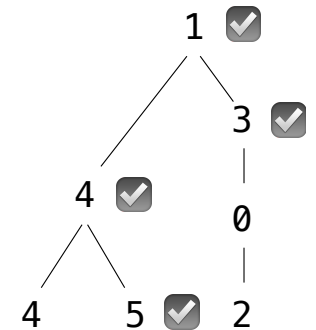
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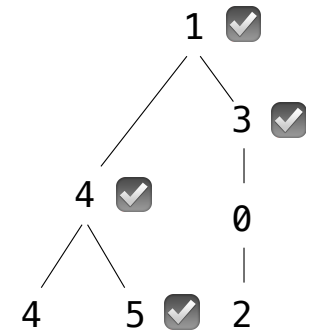
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    if node.label > max(ancestors):
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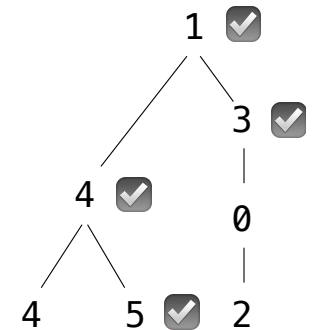
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Somehow track a
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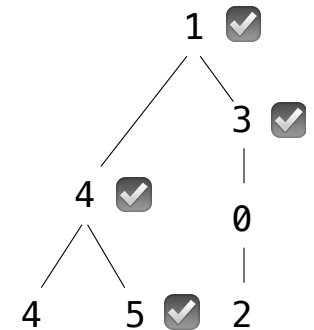
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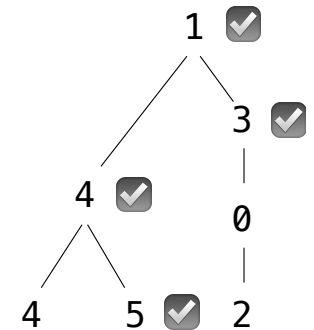
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Somehow increment
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Somehow track the
largest ancestor

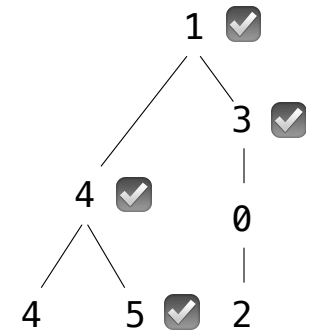
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    def f(a, x):  
        if _____:  
            return 1 + _____  
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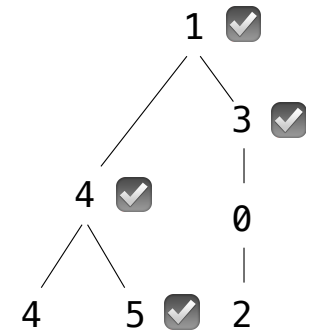
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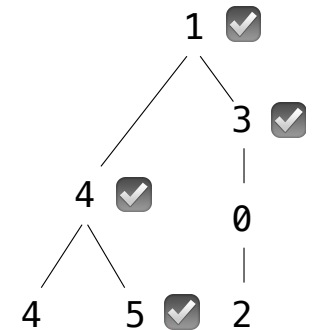
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Somehow track the largest ancestor

node.label > max_ancestors



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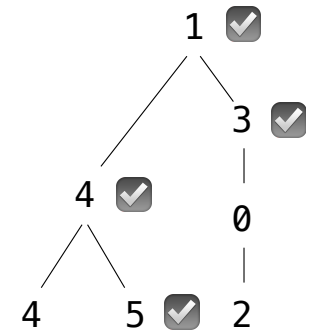
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Somehow track the largest ancestor

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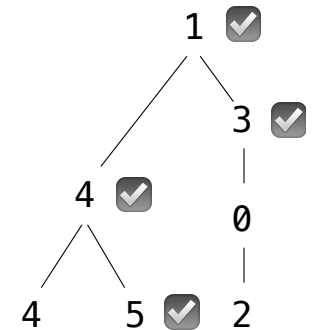
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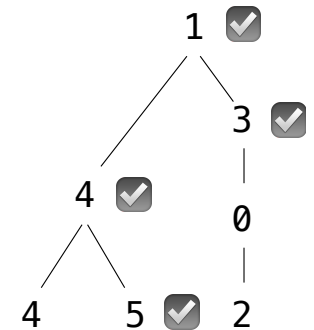
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    return f(t, _____)
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Somehow track the largest ancestor

A node in *t* \nearrow \nearrow *max_ancestor* \nwarrow node.label > max_ancestors



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```
    """
```

```
    def f(a, x):
```

```
    A node in t
```

```
    if a.label > x
```

```
        node.label > max_ancestors
```

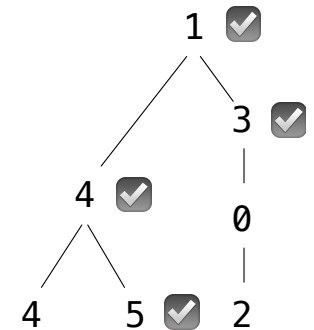
```
        return 1 +
```

```
    else:
```

```
        return
```

```
    return f(t,
```

```
        Some initial value for the largest ancestor so far...
```



Solving Tree Problems

Implement **big**s, which takes a Tree instance *t* containing integer labels. It returns the number of nodes in *t* whose labels are larger than any labels of their ancestor nodes.

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def bigs(t):
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    """Return the number of nodes in t that are larger than all their ancestors.
```

```
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```

```
    >>> bigs(a)
```

```
    4
```

```
    """
```

```
    def f(a, x):
```

```
    A node in t
```

```
    max_ancestor
```

```
    if
```

```
    a.label > x
```

```
    node.label > max_ancestors
```

```
:
```

```
        return 1 +
```

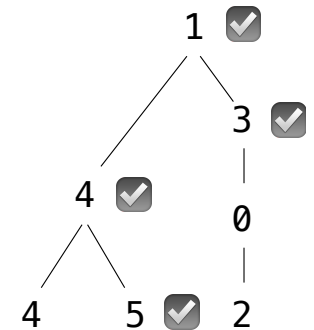
```
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        return
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```

```
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node.label > max_ancestors

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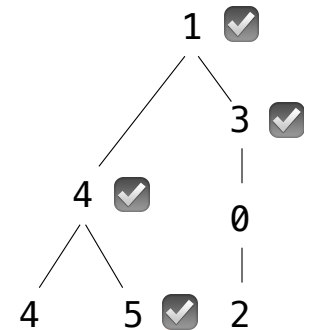
```
        else:
```

Somehow increment the total count

```
            return
```

```
    return f(t,
```

Some initial value for the largest ancestor so far...



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```
    4
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```
:
```

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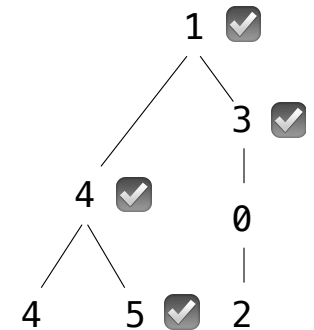
```
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```

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```

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```
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```

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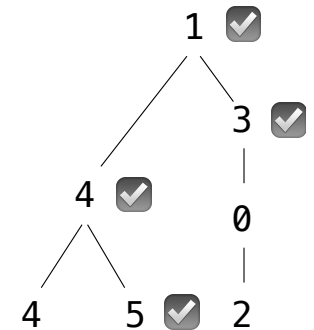
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```
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```

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```

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



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```
:
```

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    return 1 + sum([f(b, a.label) for b in a.branches])
```

```
    else:
```

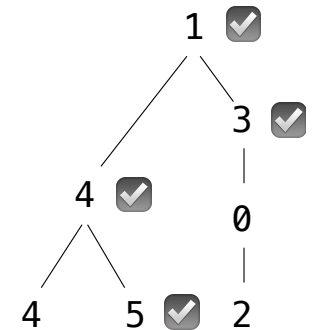
```
    Somehow increment the total count
```

```
    return sum([f(b, x) for b in a.branches])
```

```
    return f(t, t.label - 1)
```

```
    Root label is always larger than its ancestors
```

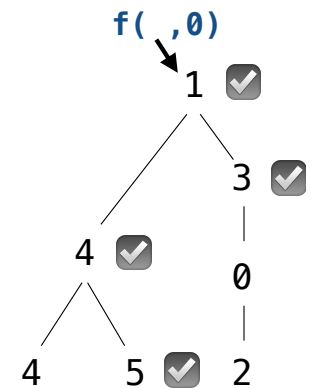
```
    Some initial value for the largest ancestor so far...
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Solving Tree Problems

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        A node in t
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        if a.label > x
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```
            max_ancestor
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```
            node.label > max_ancestors
```

```
        :
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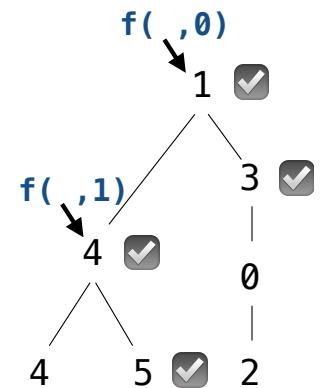
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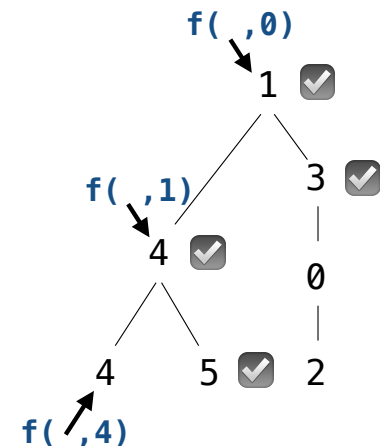
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Root label is always larger than its ancestors

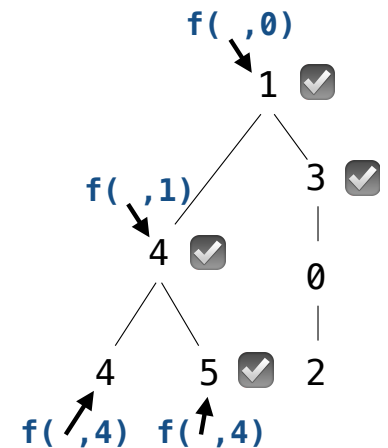
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A node in t

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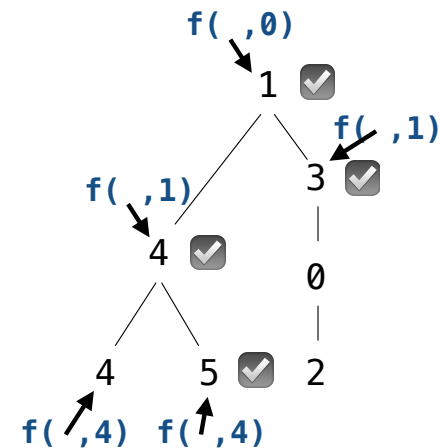
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Root label is always larger than its ancestors

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```
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```
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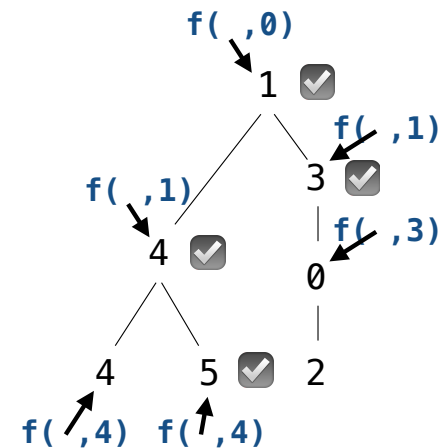
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```
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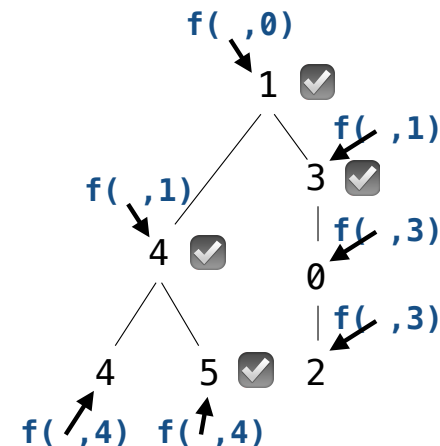
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Root label is always larger than its ancestors

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```
:
```

```
        return 1 + sum( f(b, a.label) for b in a.branches )
```

```
    else:
```

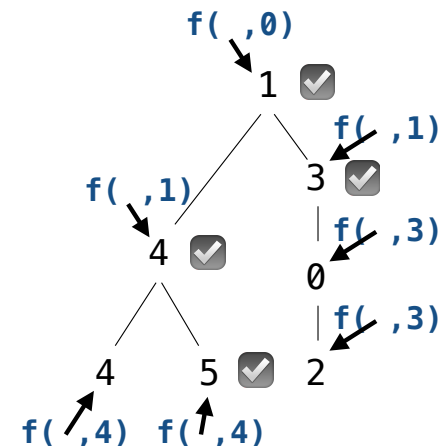
Somehow increment the total count

```
        return sum( f(b, x) for b in a.branches )
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    return f(t, t.label - 1)
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Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Recursive Accumulation

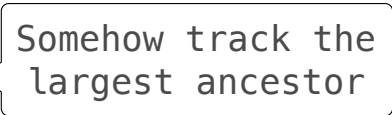
Solving Tree Problems

Implement `big`, which takes a Tree instance `t` containing integer labels. It returns the number of nodes in `t` whose labels are larger than any labels of their ancestor nodes.

```
def big(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = 0  
  
    def f(a, x):  
        _____  
  
        if _____:  
            n += 1  
  
        _____:  
            f(_____)  
  
        _____  
  
    return n
```

Solving Tree Problems

Implement **big**s, which takes a Tree instance *t* containing integer labels. It returns the number of nodes in *t* whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):  
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    n = 0  
    def f(a, x):  
        Somehow track the  
        largest ancestor  
        _____  
        if _____:  
            n += 1  
        _____:  
            f(_____)  
        _____  
    return n
```

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```

```
    n = 0
```

```
    def f(a, x):
```

Somehow track the
largest ancestor

```
        if _____:
```

node.label > max_ancestors

```
            n += 1
```

```
        _____:
```

```
            f(_____)
```

```
    return n
```

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Somehow increment
the total count

```
            f(_____)
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Somehow track the
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```
        if
```

node.label > max_ancestors

```
            n += 1
```

Somehow increment
the total count

```
            f(
```

Root label is always larger than its ancestors

```
    return n
```

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```
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```

Somehow track the
largest ancestor

```
        if a.label > x:
```

node.label > max_ancestors

```
            n += 1
```

Somehow increment
the total count

```
        f(a.left, a.label)
```

Root label is always larger than its ancestors

```
    return n
```


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```
            n += 1
```

Somehow increment
the total count

```
            f(a.left, a.label)
```

```
    f(t, t.label - 1)
```

Root label is always larger than its ancestors

```
    return n
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```
            n += 1
```

```
        for b in a.branches:
```

Somehow increment
the total count

```
            f(_____)
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            n += 1
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```
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```

Somehow increment
the total count

```
            f(b, max(a.label, x))
```

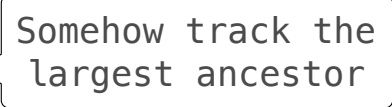

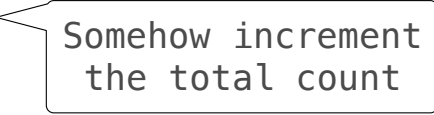

```
    f(t, t.label - 1)
```

Root label is always larger than its ancestors

```
    return n
```

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Implement **big**s, which takes a Tree instance *t* containing integer labels. It returns the number of nodes in *t* whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):  
    """Return the number of nodes in t that are larger than all their ancestors."""  
    n = 0  
    def f(a, x):  
         Somehow track the largest ancestor  
        nonlocal n  
         node.label > max_ancestors  
        if a.label > x:  
            n += 1  
             Somehow increment the total count  
        for b in a.branches:  
            f(b, max(a.label, x))  
    f(t, t.label - 1)  Root label is always larger than its ancestors  
    return n
```

Designing Functions

How to Design Programs

<https://htdp.org/2018-01-06/Book/>

How to Design Programs

From Problem Analysis to Data Definitions

Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and illustrate them with examples.

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Signature, Purpose Statement, Header

State what kind of data the desired function consumes and produces. Formulate a concise answer to the question *what* the function computes. Define a stub that lives up to the signature.

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Applying the Design Process

Designing a Function

Implement `smalls`, which takes a `Tree` instance `t` containing integer labels. It returns the non-leaf nodes in `t` whose labels are smaller than any labels of their descendant nodes.

```
def smalls(t):  
    """Return the non-leaf nodes in t that are smaller than all their descendants.  
  
    >>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])  
    >>> sorted([t.label for t in smalls(a)])  
    [0, 2]  
    """  
    result = []  
    def process(t):  
  
        process(t)  
    return result
```

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Implement `smalls`, which takes a `Tree` instance `t` containing integer labels. It returns the non-leaf nodes in `t` whose labels are smaller than any labels of their descendant nodes.

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```
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```

```
[0, 2]
```

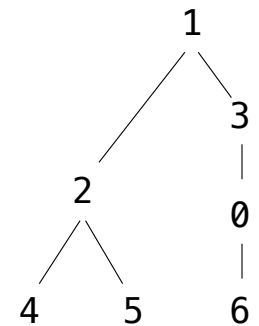
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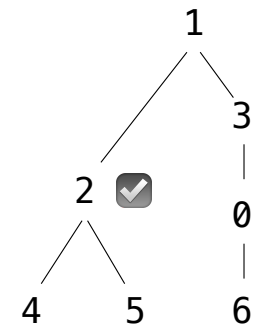
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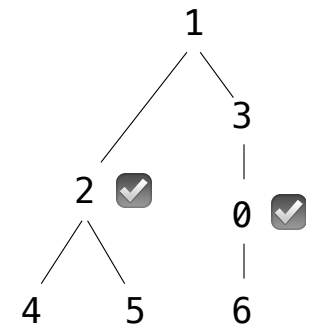
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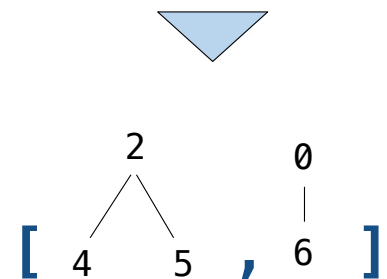
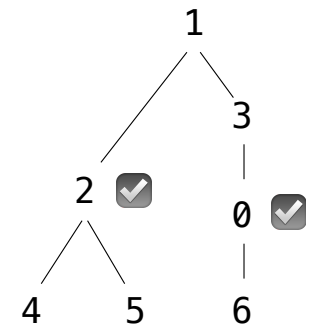
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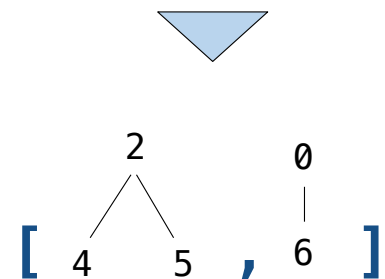
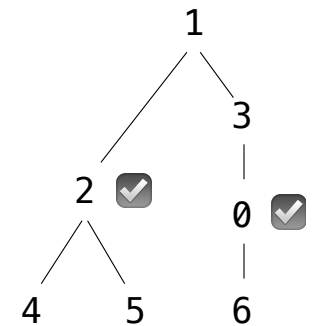
Signature: Tree -> number

```
result = []
```

```
def process(t):
```

```
    process(t)
```

```
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```



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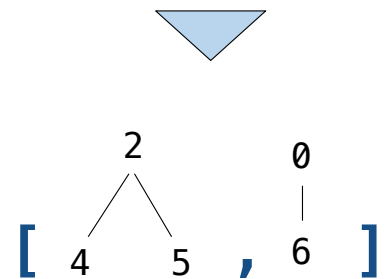
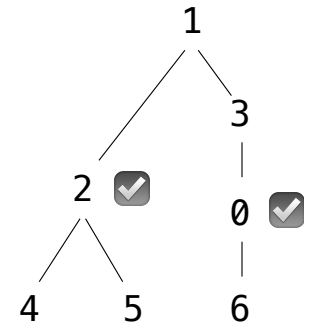
```
result = []
```

```
def process(t):     Signature: Tree -> number
```

"Find smallest label in t & maybe add t to result"

```
process(t)
```

```
return result
```



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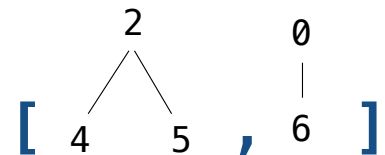
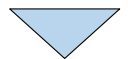
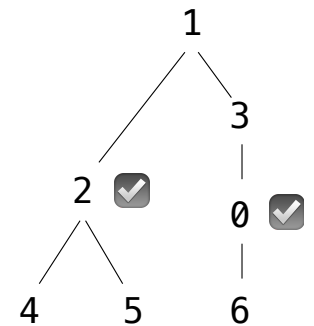
"""

Signature: Tree -> number

def process(t): "Find smallest label in t & maybe add t to result"

```
        if t.is_leaf():
            return t.label
        else:
```

```
            return min(...)
    process(t)
    return result
```



Designing a Function

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result = [] *Signature: Tree -> number*

def process(t): *"Find smallest label in t & maybe add t to result"*

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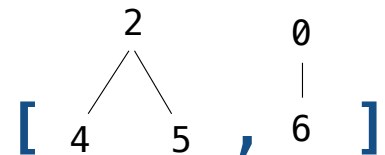
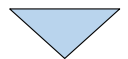
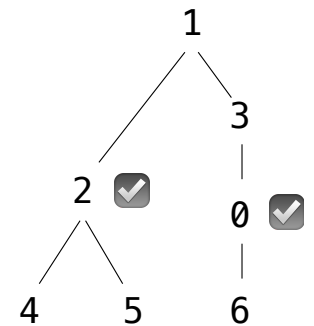
 smallest = _____

if _____:

return min(smallest, t.label)

 process(t)

return result



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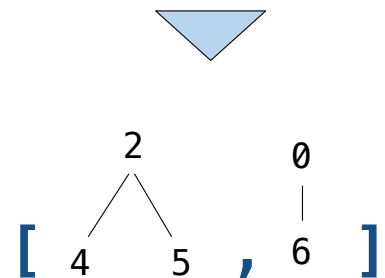
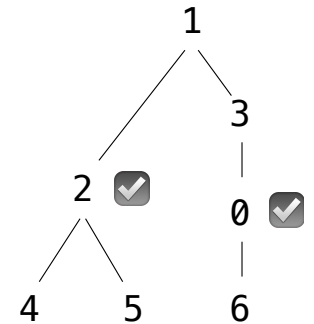
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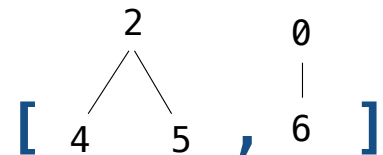
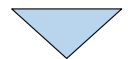
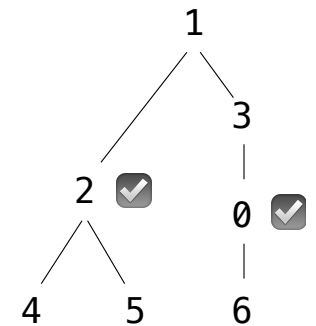
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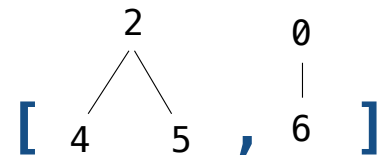
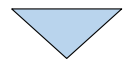
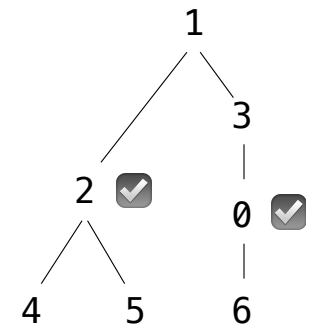
smallest = _____

if _____ `t.label < smallest` _____ :

return `min(smallest, t.label)`

process(t)

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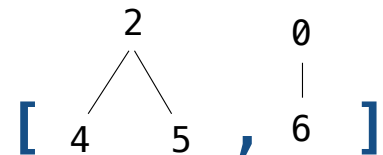
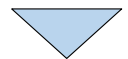
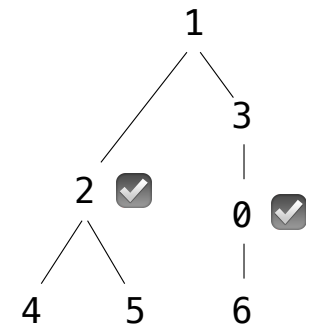
if _____ `t.label < smallest`

result.append(_____ *)*

return `min(smallest, t.label)`

process(t)

return `result`



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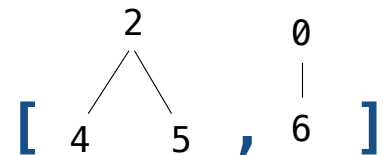
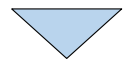
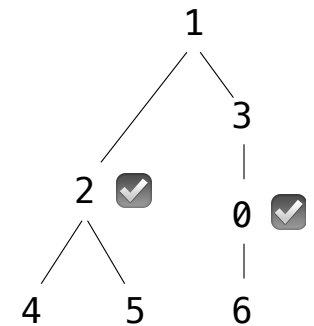
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```

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```

```
[0, 2]
```

"""

result = [] *Signature: Tree -> number*

def process(t): *"Find smallest label in t & maybe add t to result"*

if t.is_leaf():

return _____ `t.label`

else:

smallest = _____ `min([process(b) for b in t.branches])`

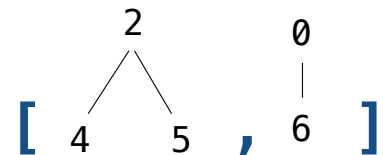
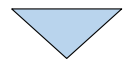
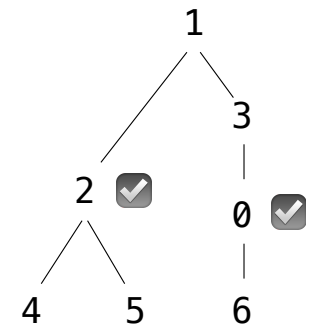
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Expression Trees

Interpreter Analysis

How many times does `scheme_eval` get called when evaluating the following expressions?

```
(define x (+ 1 2))
```

```
(define (f y) (+ x y))
```

```
(f (if (> 3 2) 4 5))
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`(define x (+ 1 2))`

`(define (f y) (+ x y))`

`(f (if (> 3 2) 4 5))`