Composition

Announcements

Linked Lists

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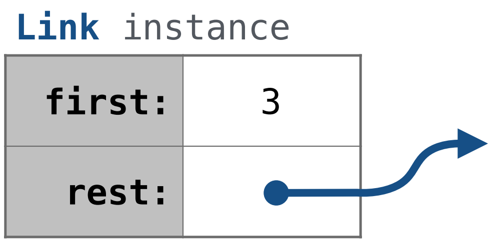

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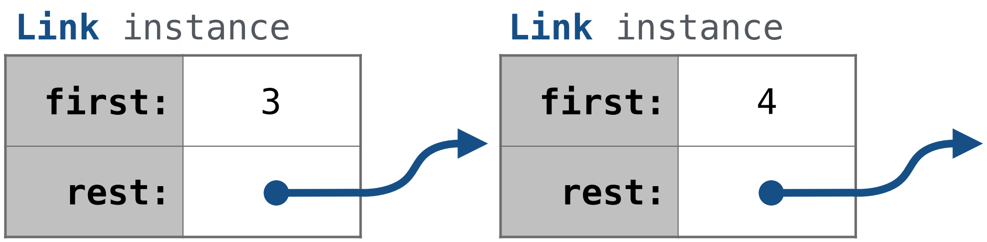

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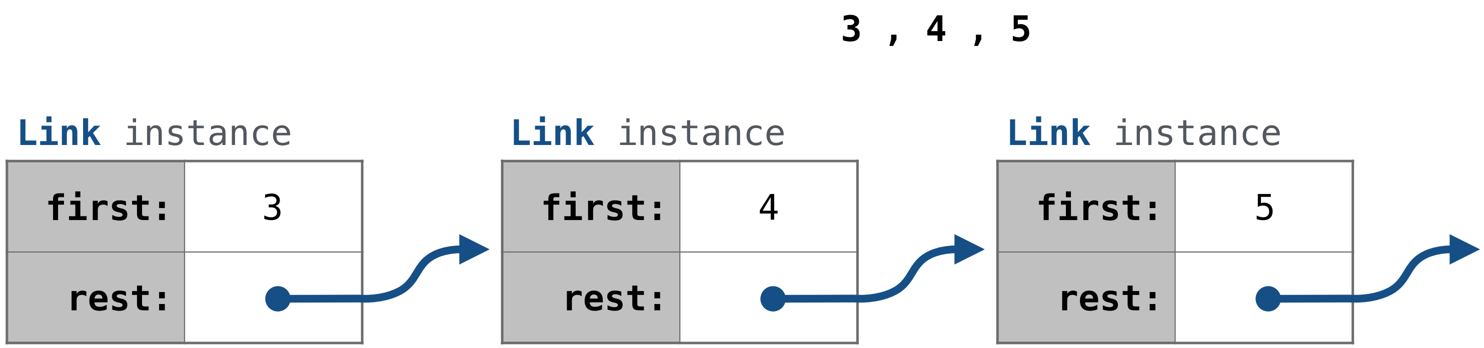

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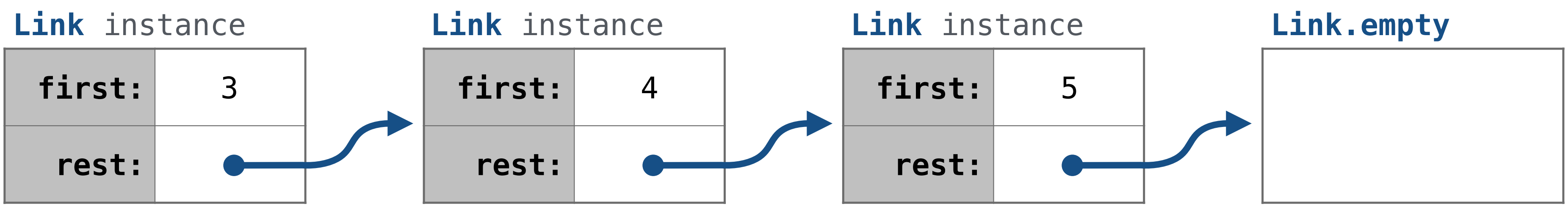

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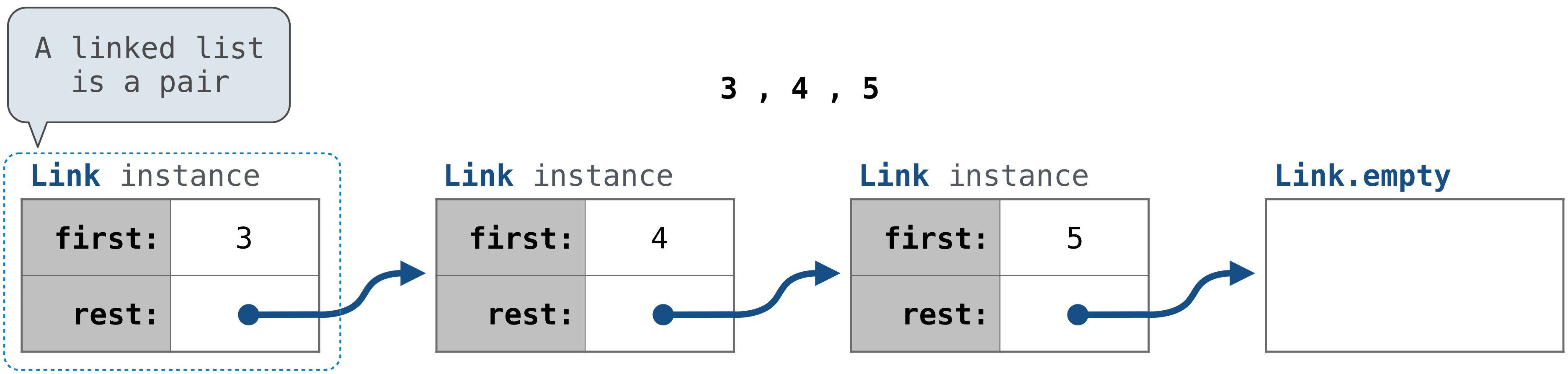

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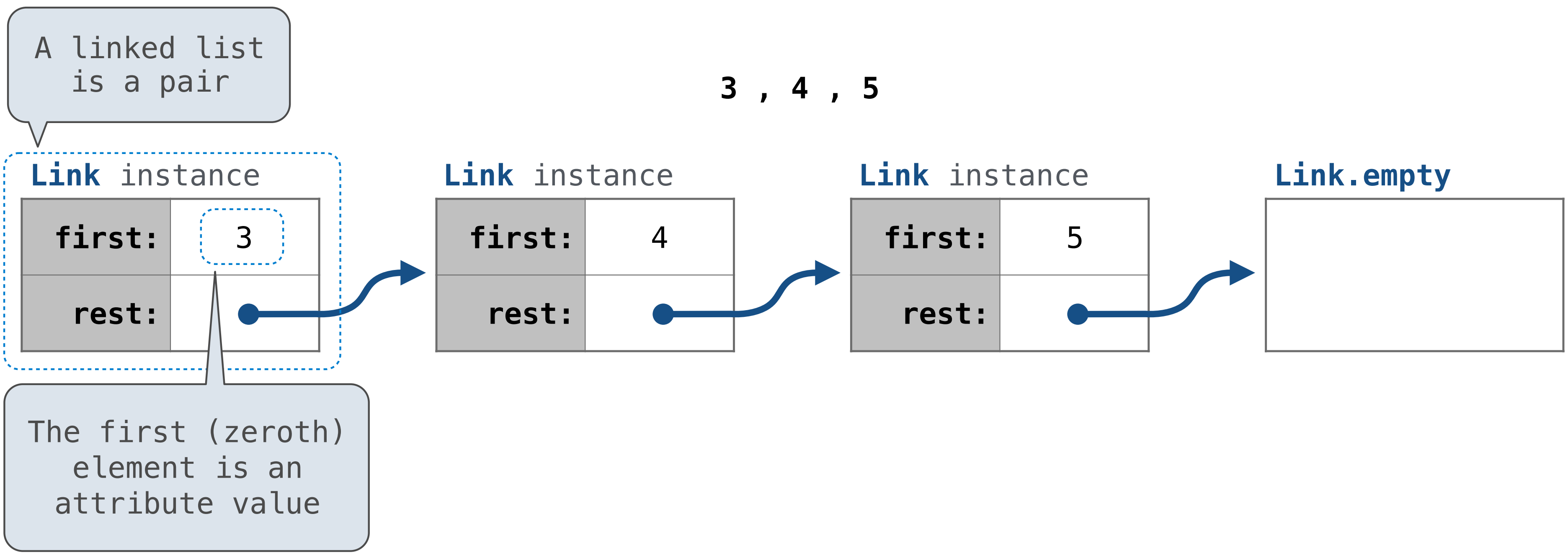

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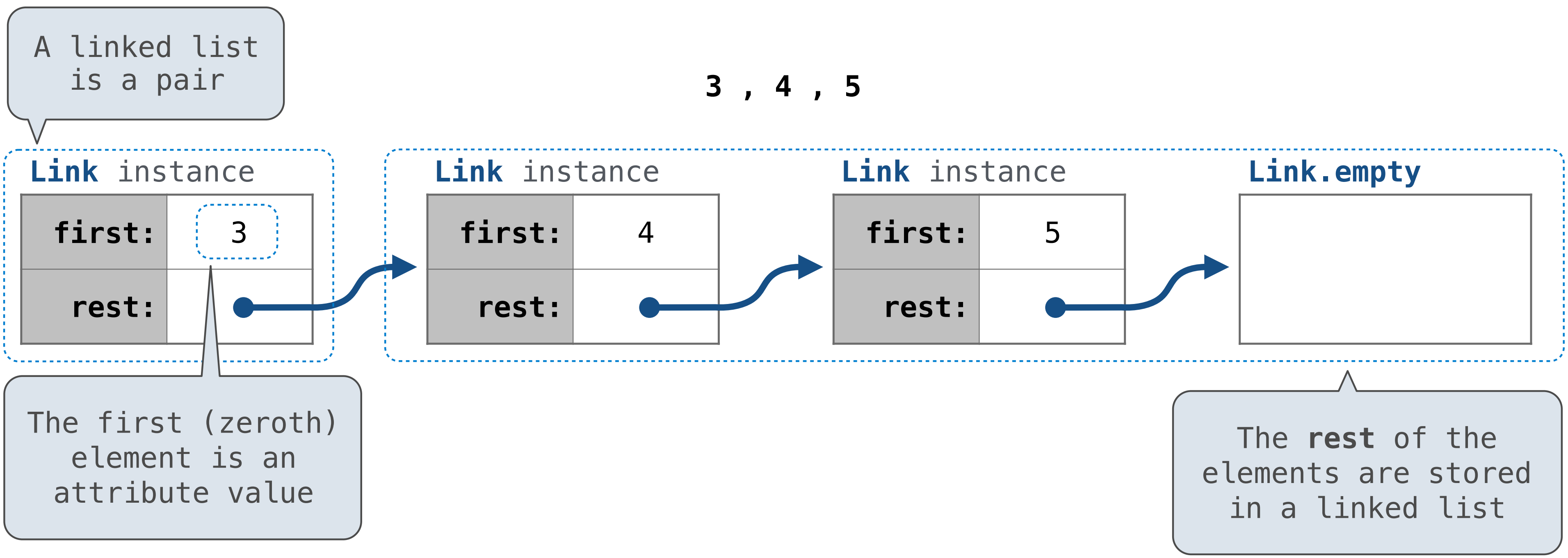

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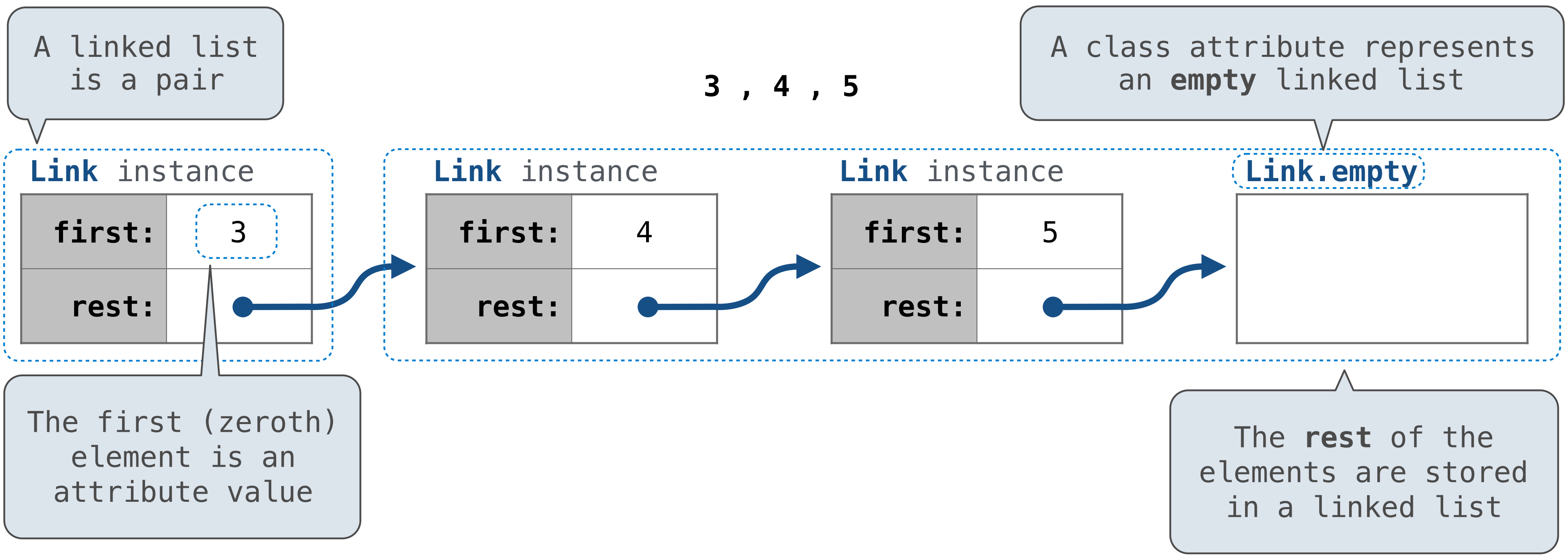

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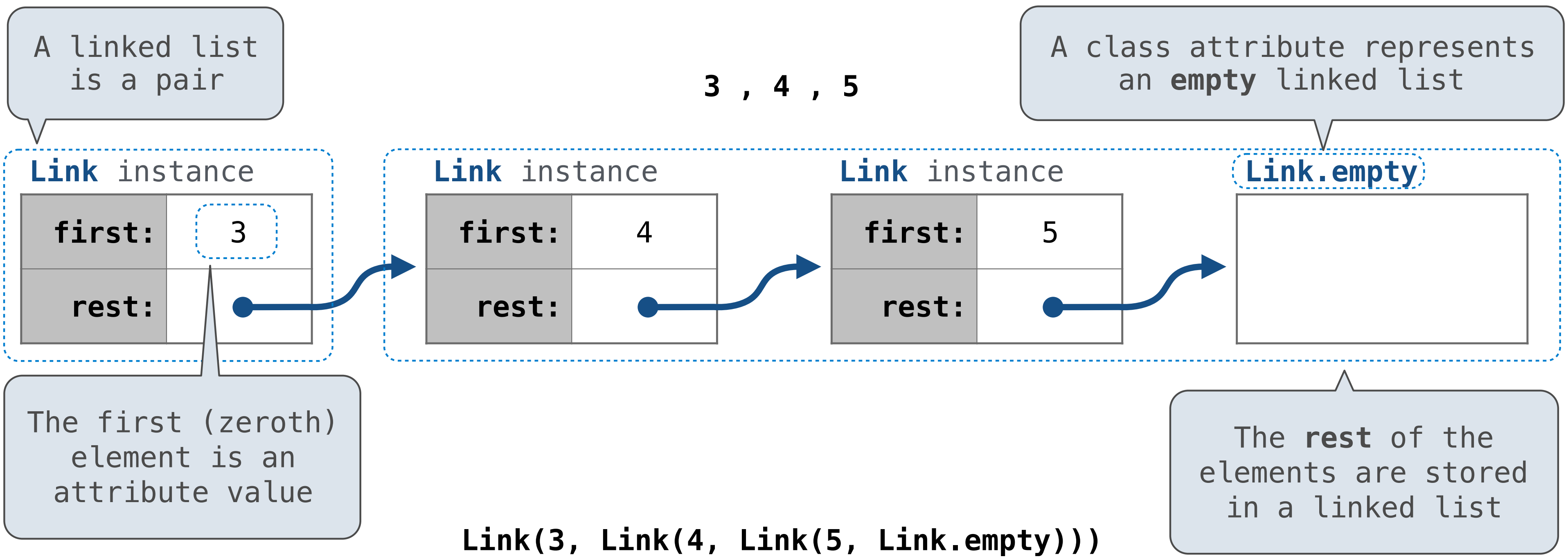

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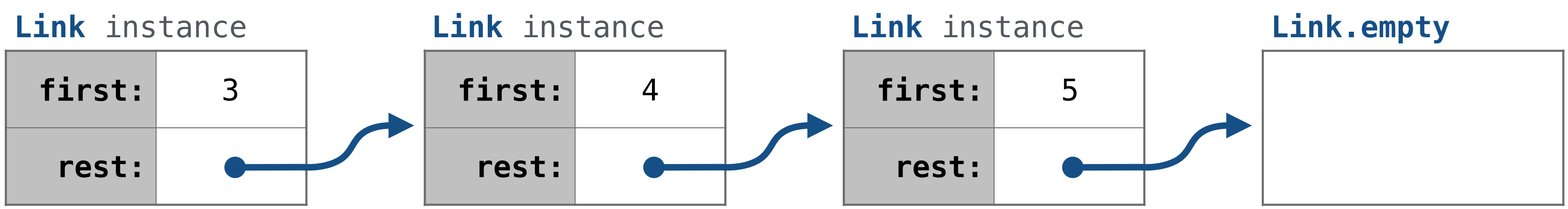


```
Link(3, Link(4, Link(5, Link.empty)))
```

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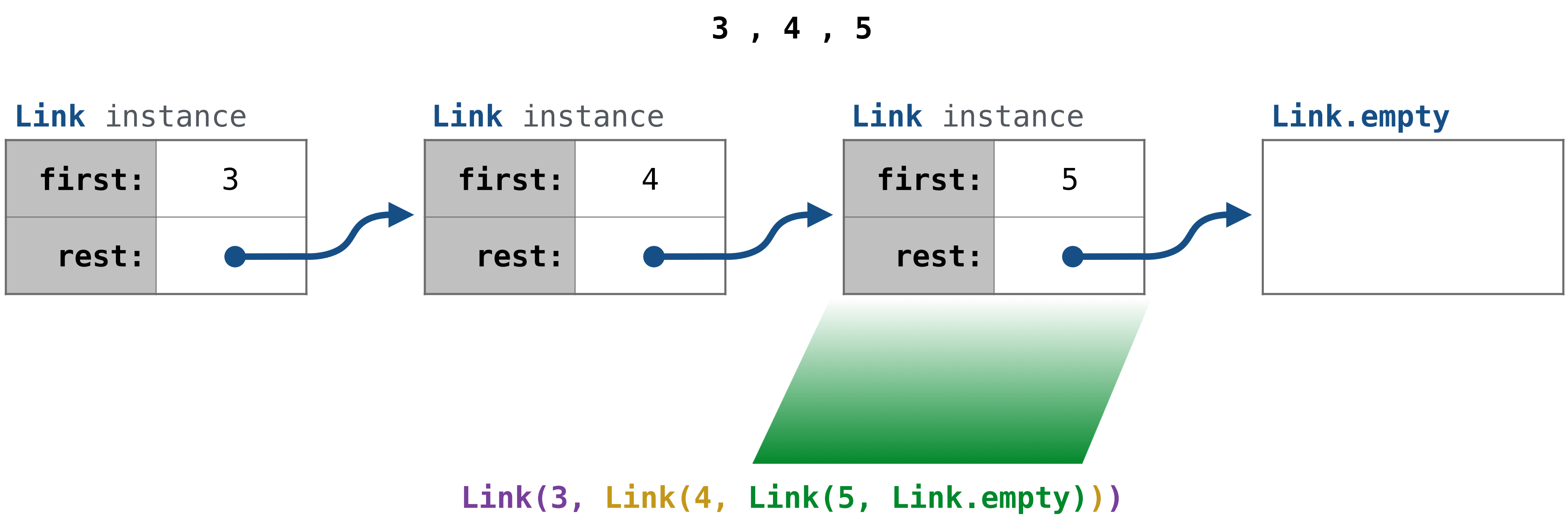

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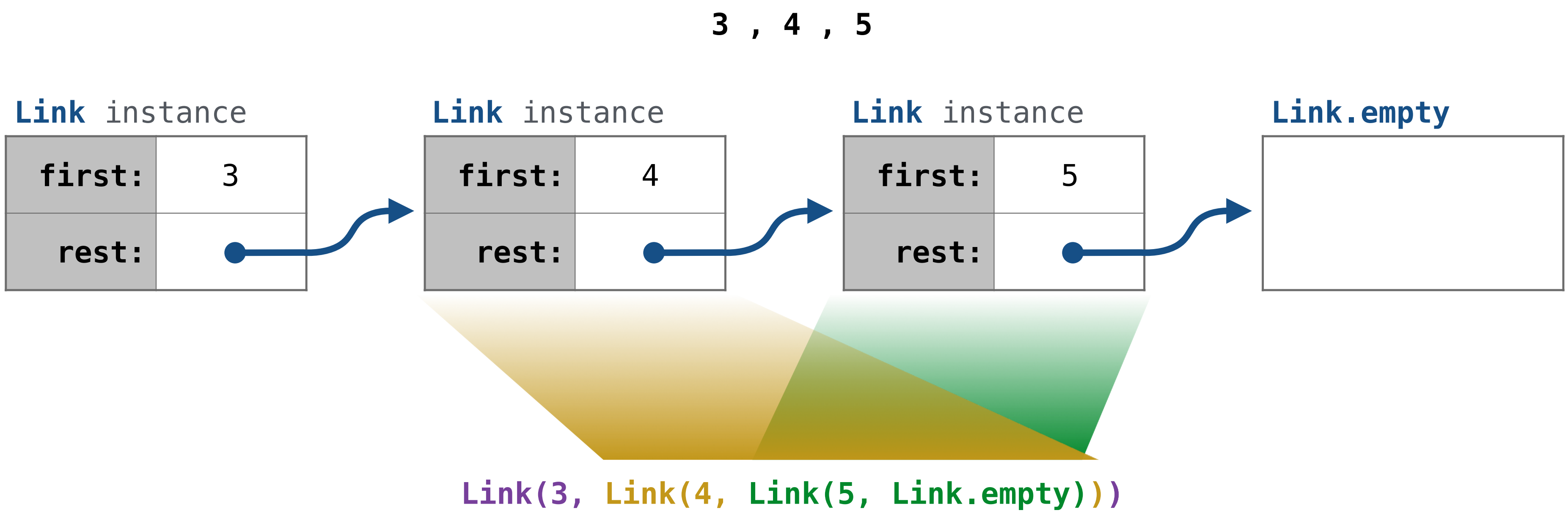

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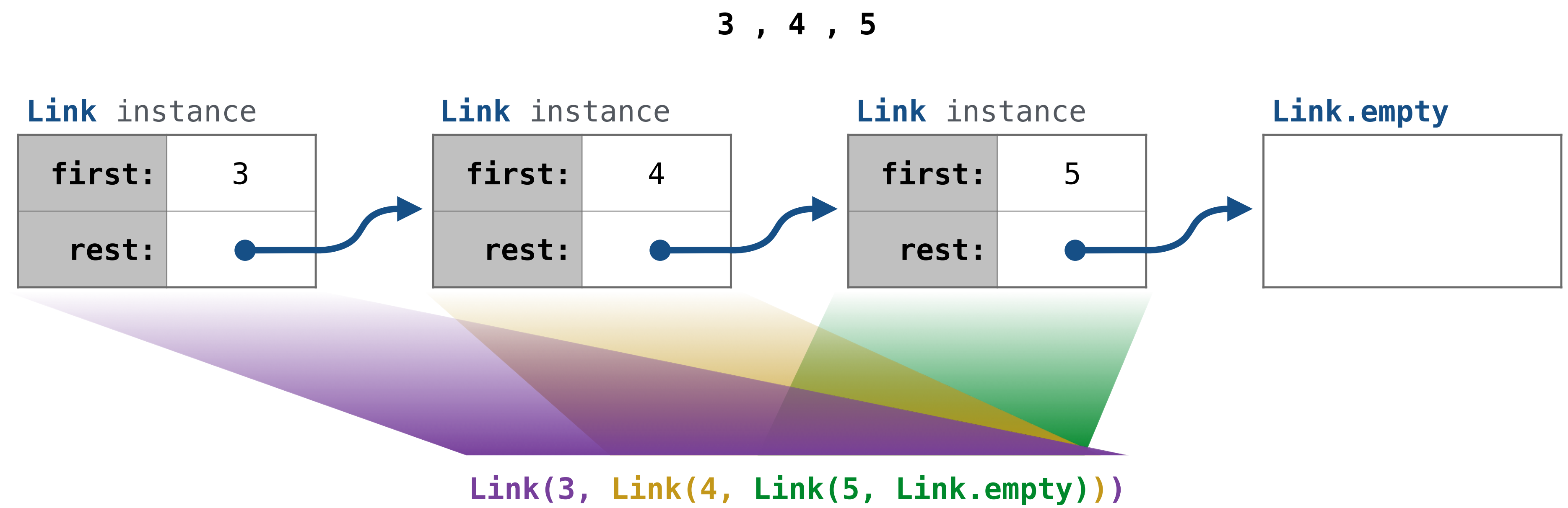

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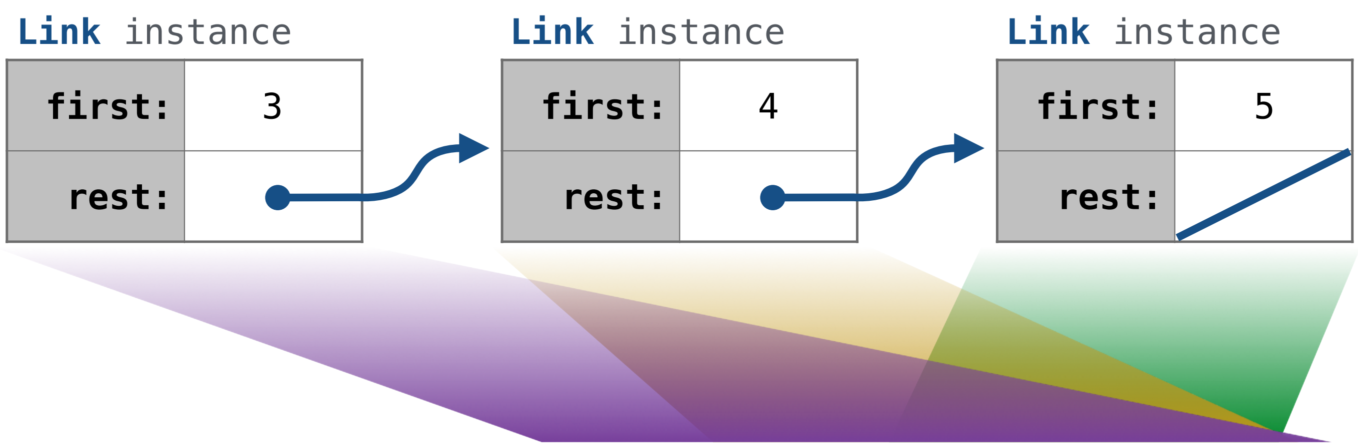

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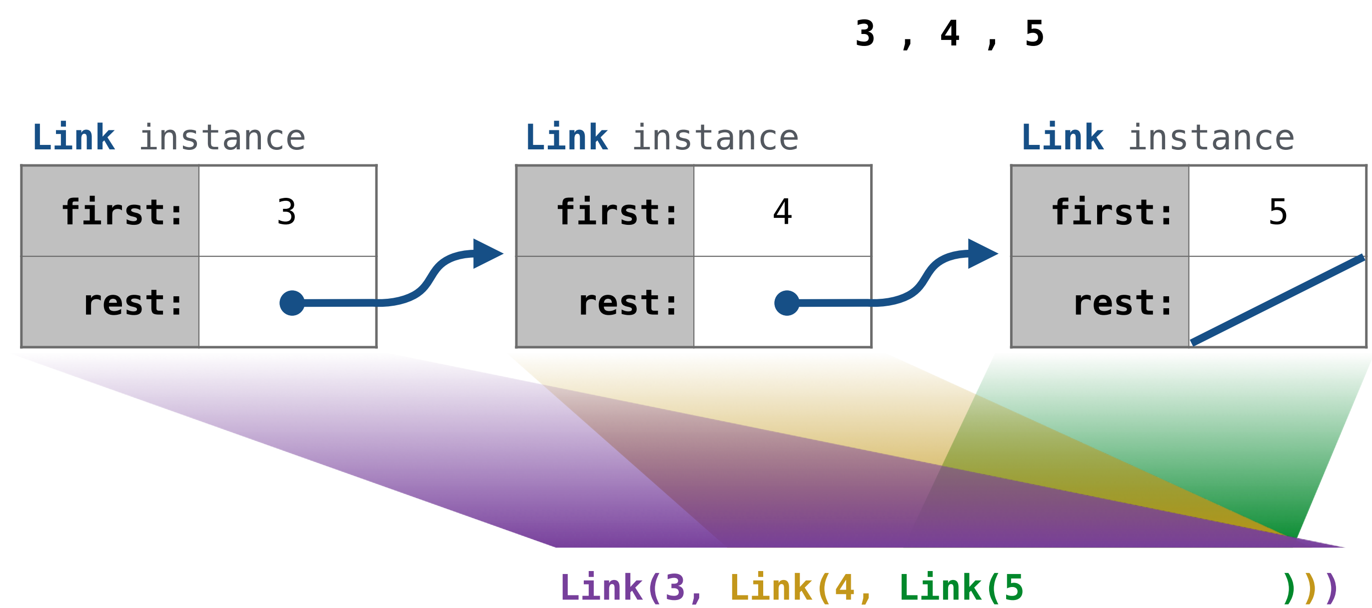

Linked List Class

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Linked list class: attributes are passed to `__init__`

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class Link:

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class Link:

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(Demo)

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```
>>> s = Link(3, Link(4, Link(5)))
>>> s.second
4
```

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(Demo)

Tree Recursion Efficiency

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

Recursive Computation of the Fibonacci Sequence

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def fib(n):
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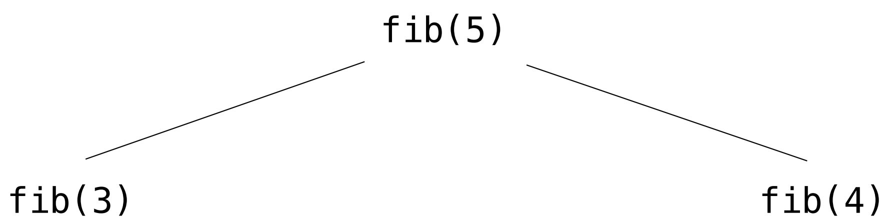


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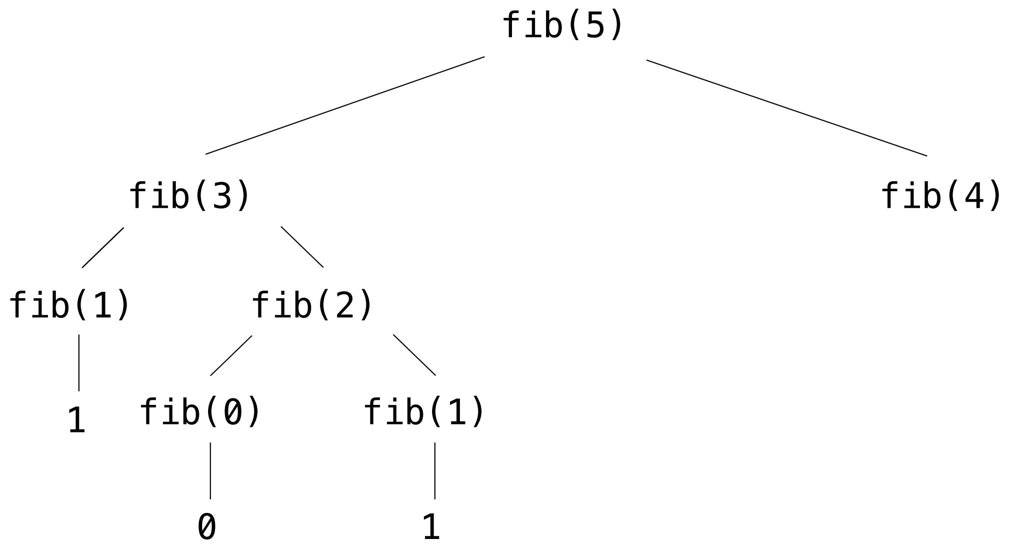


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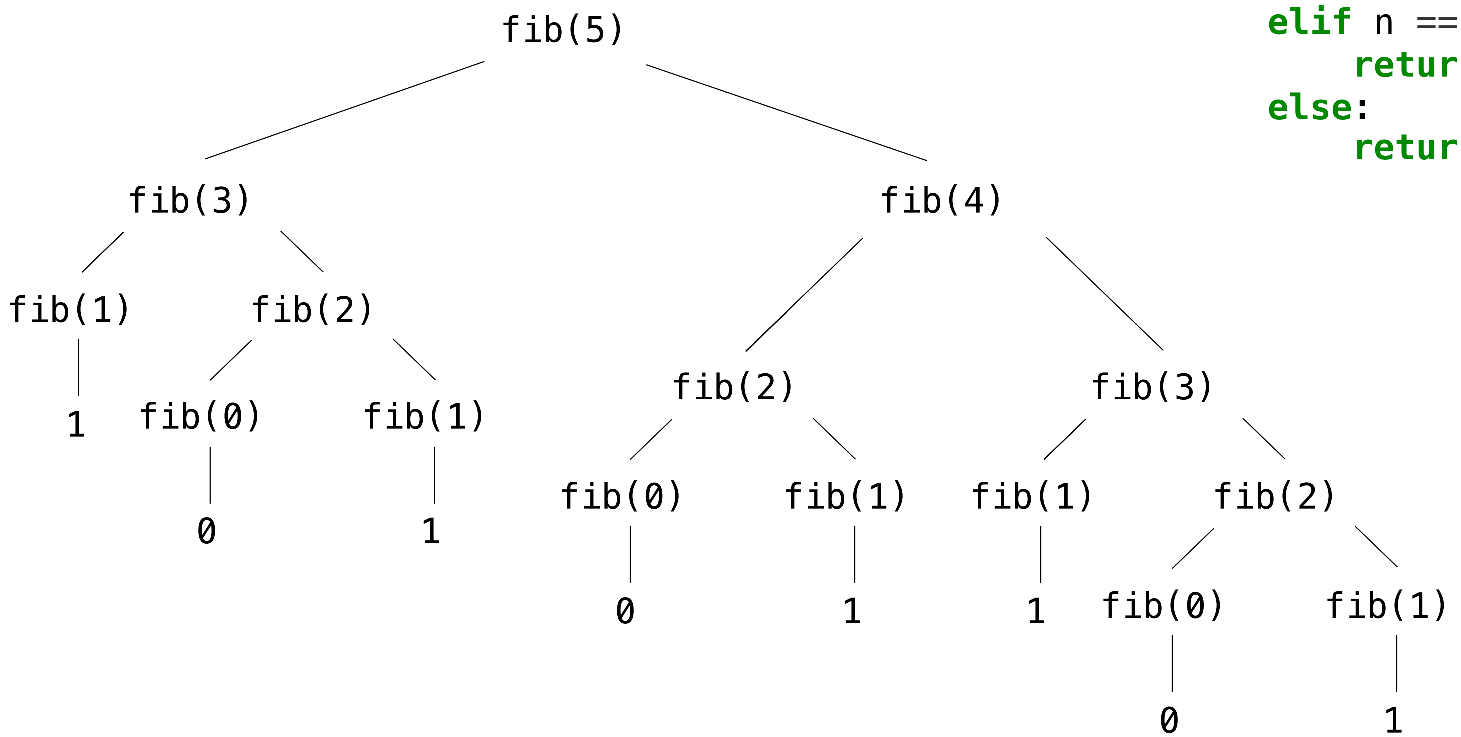

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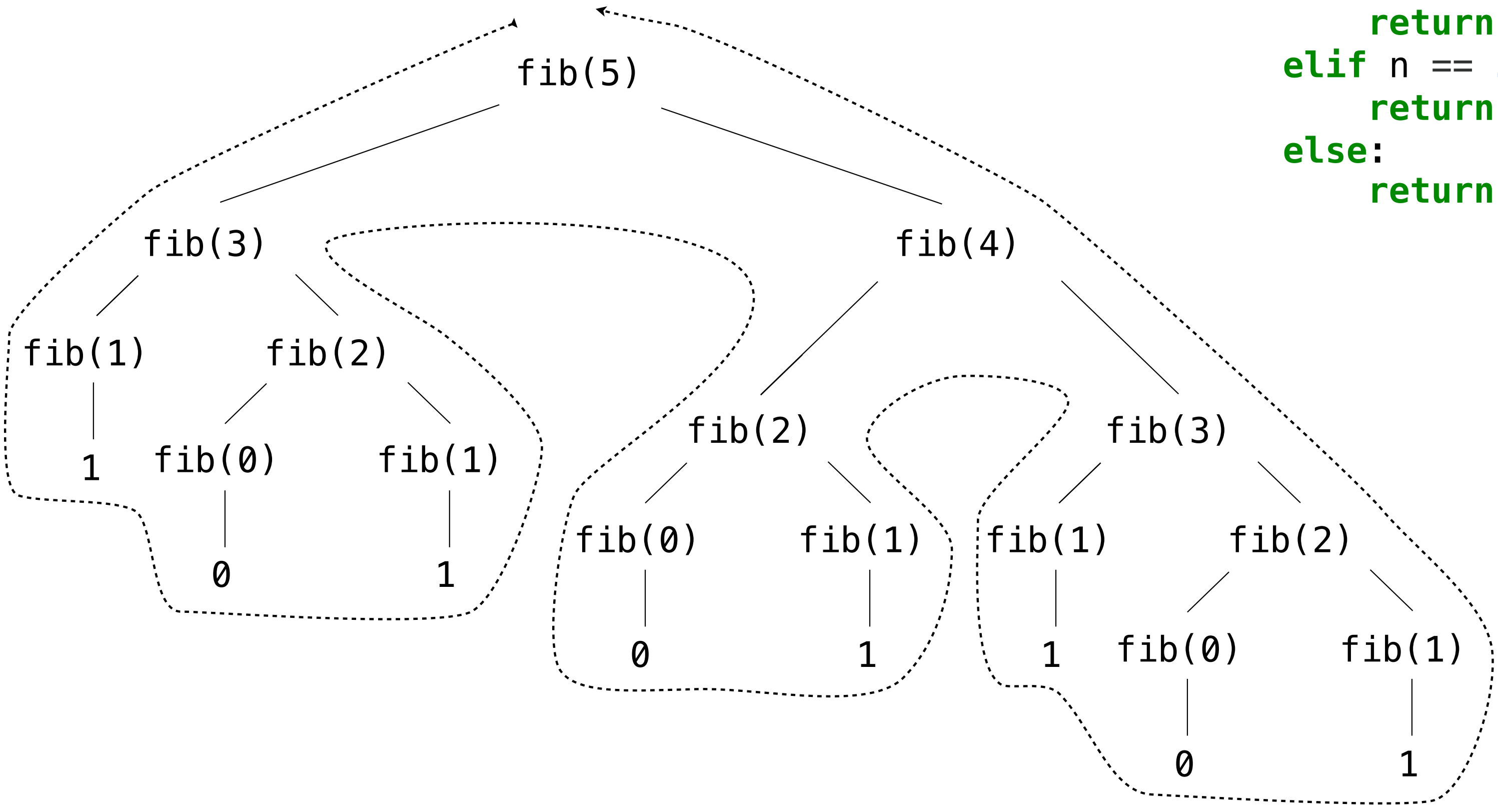

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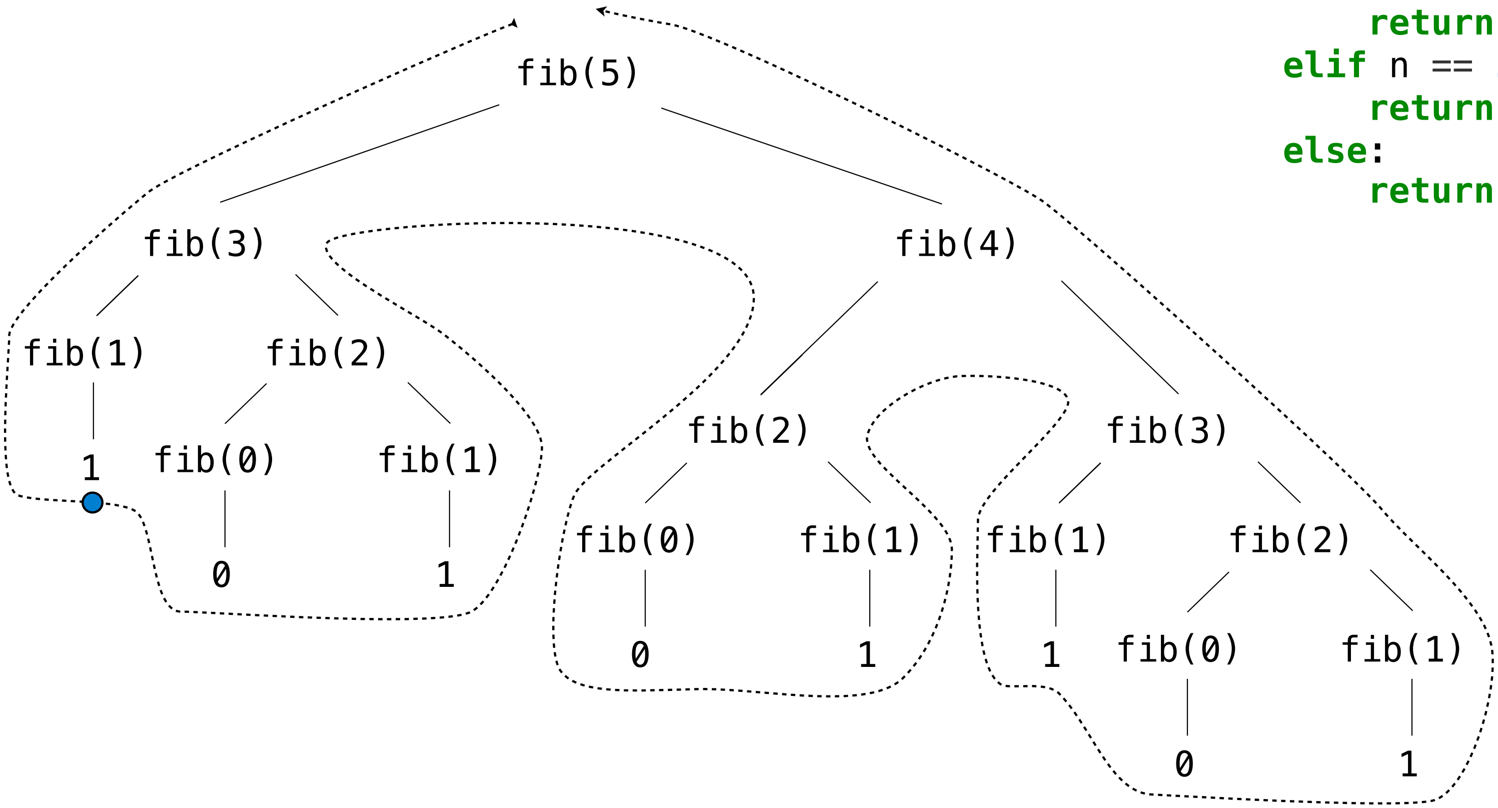

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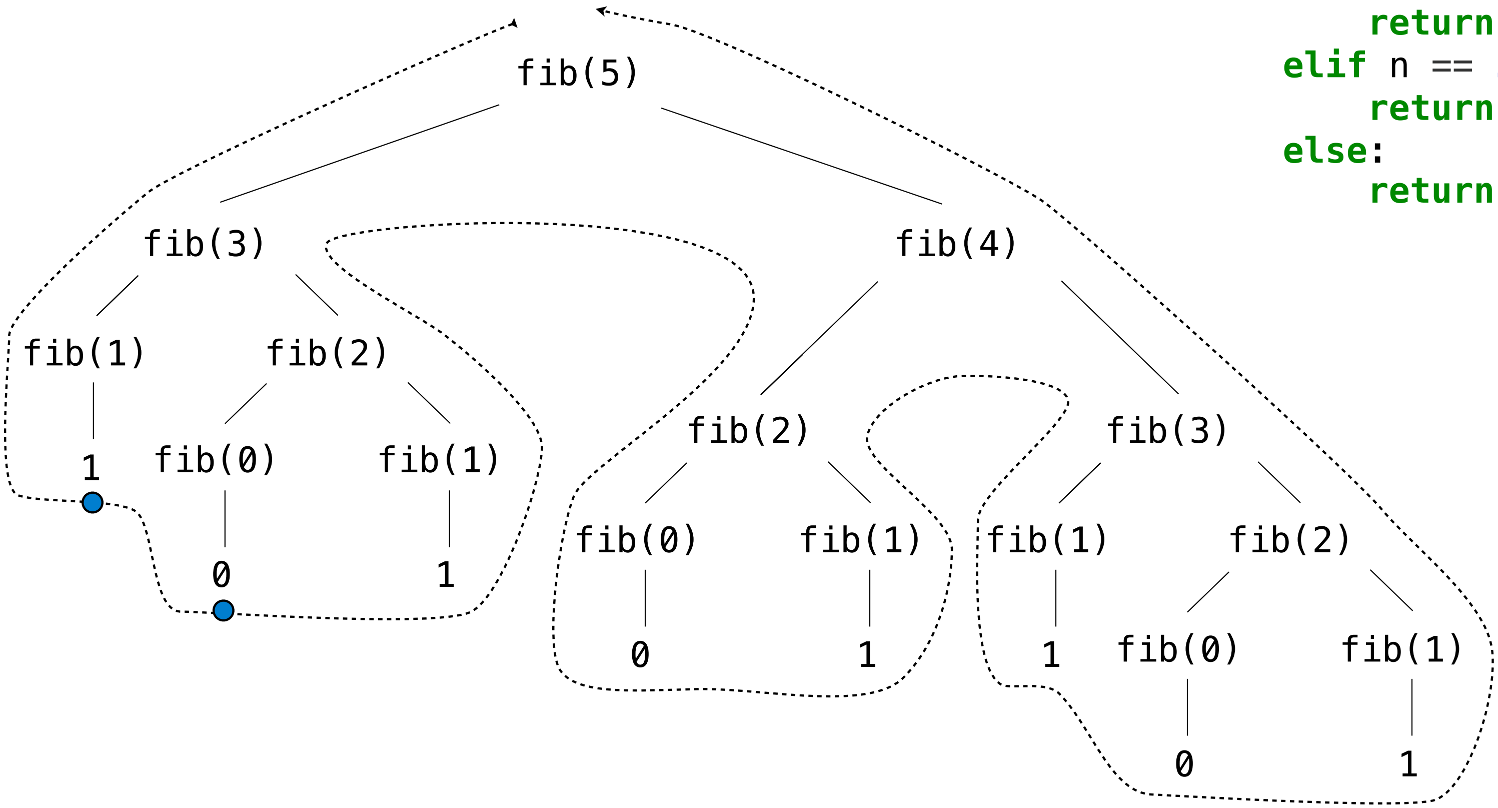

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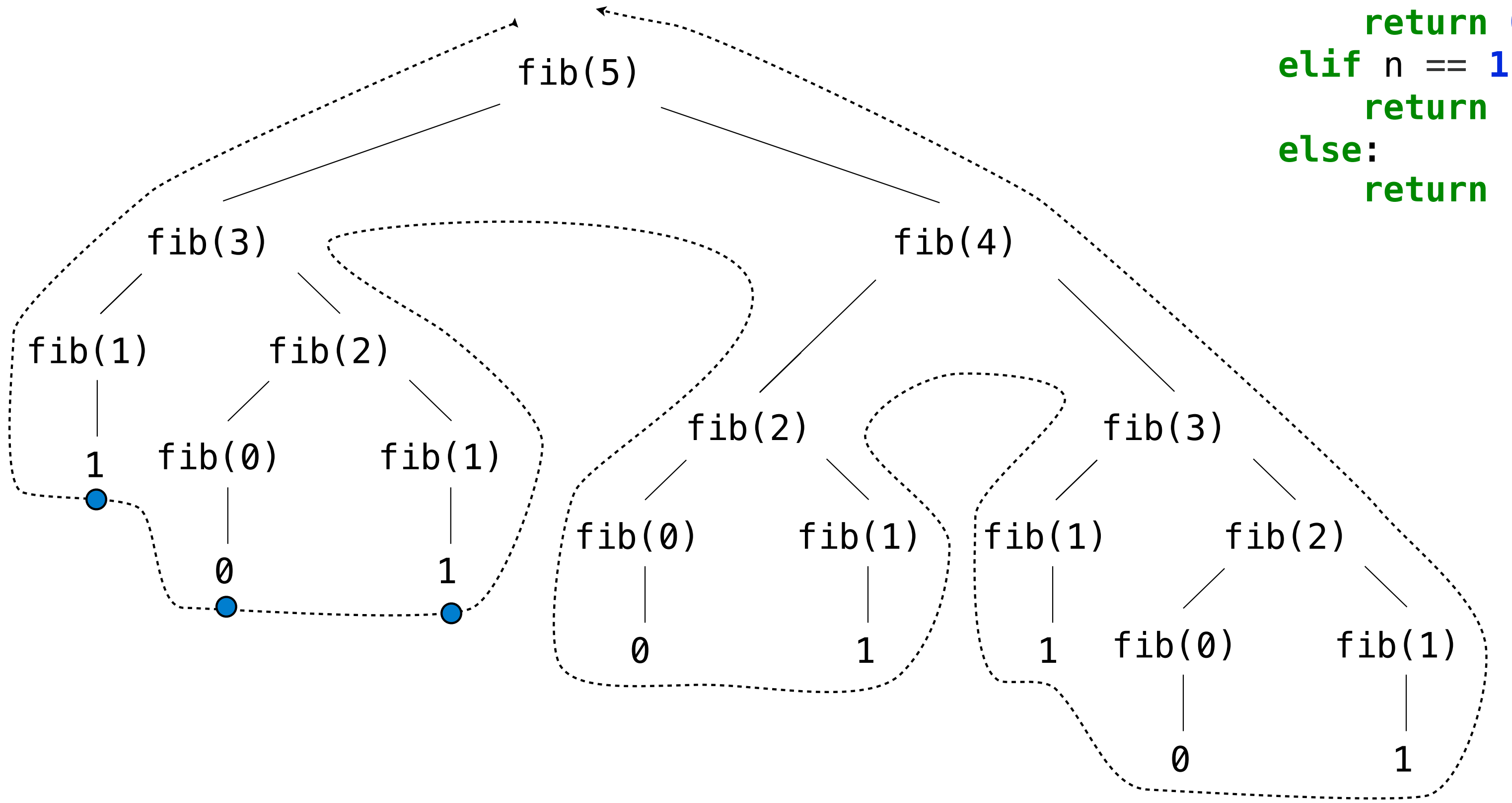


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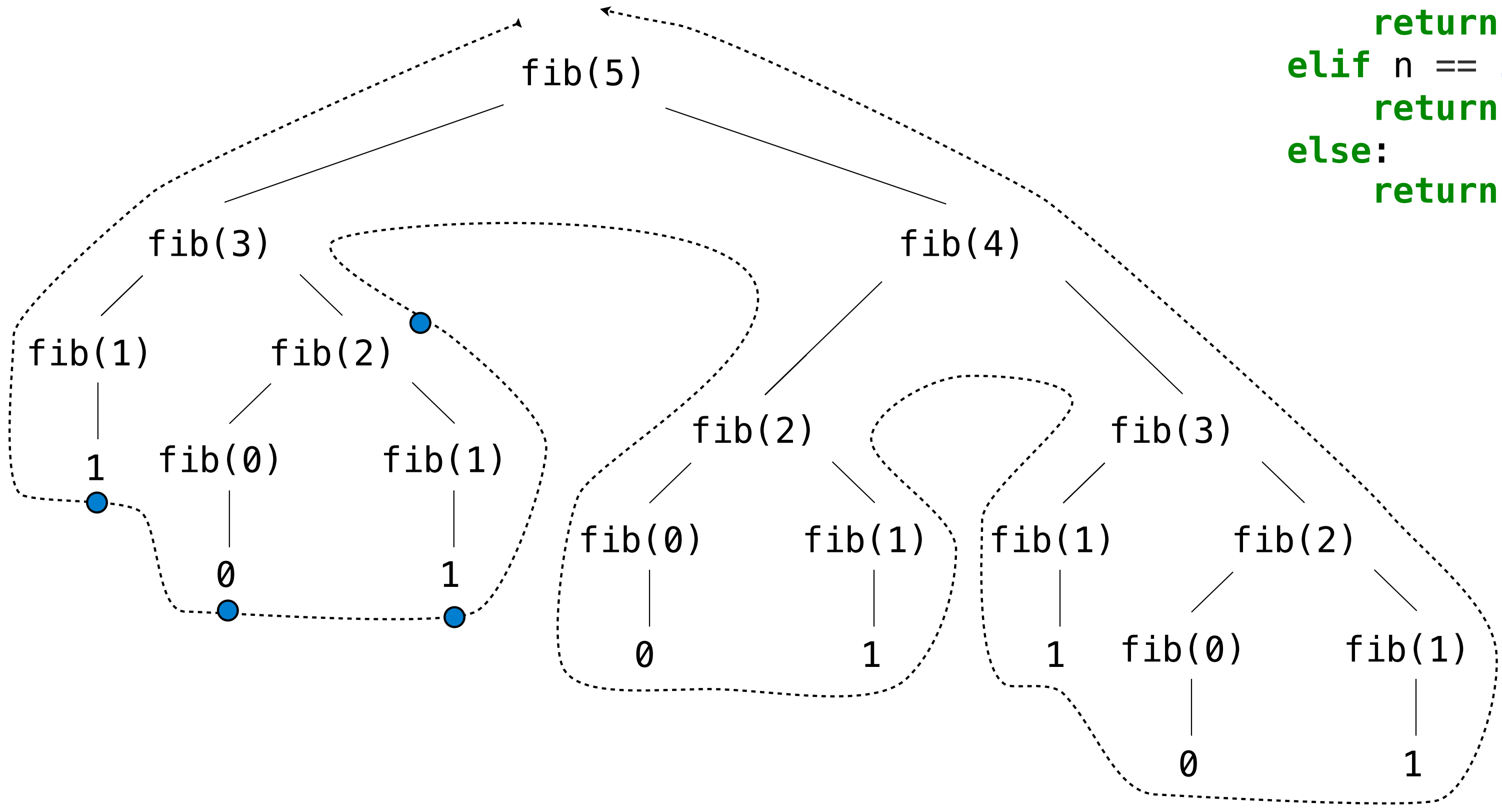

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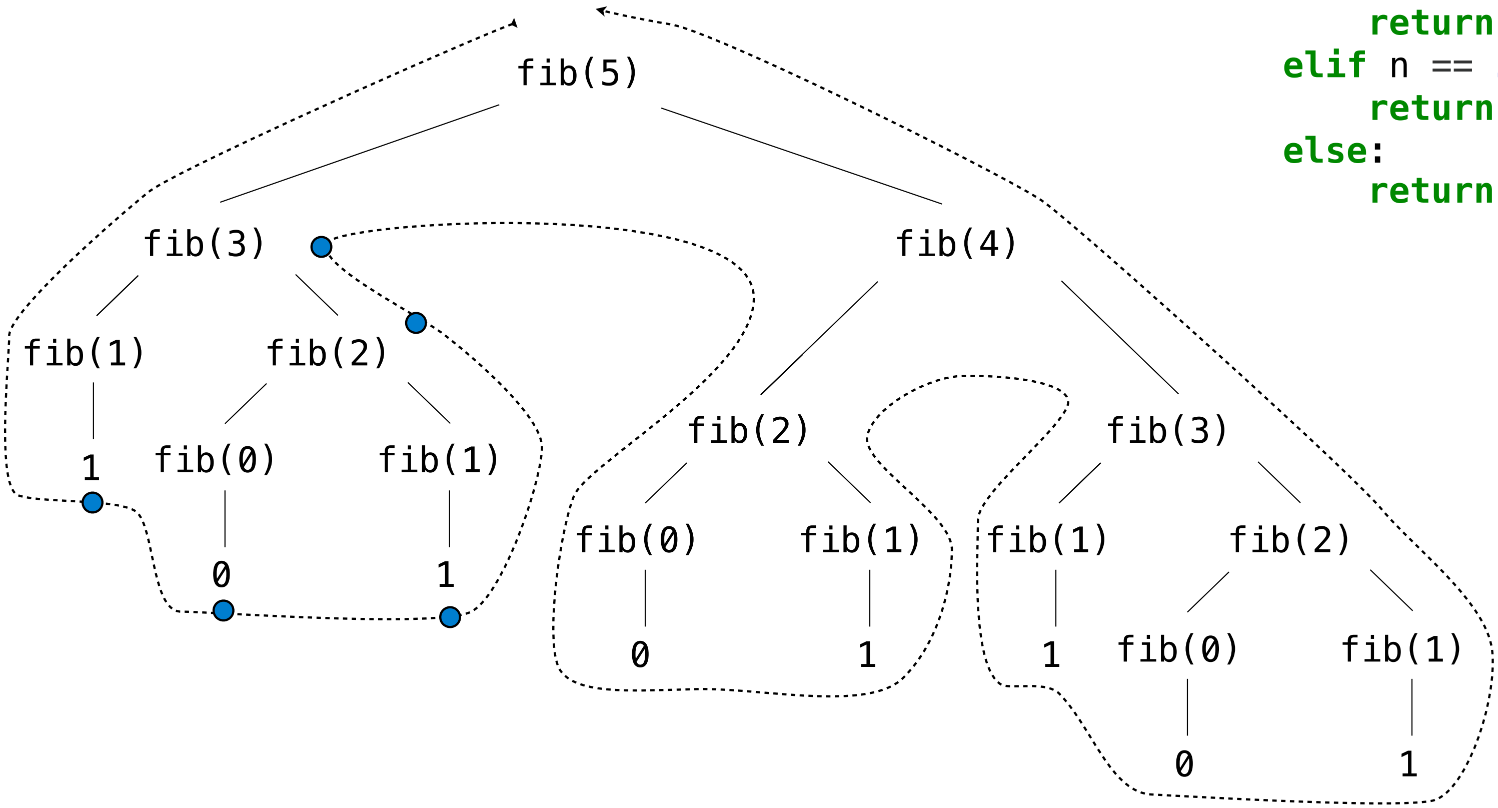

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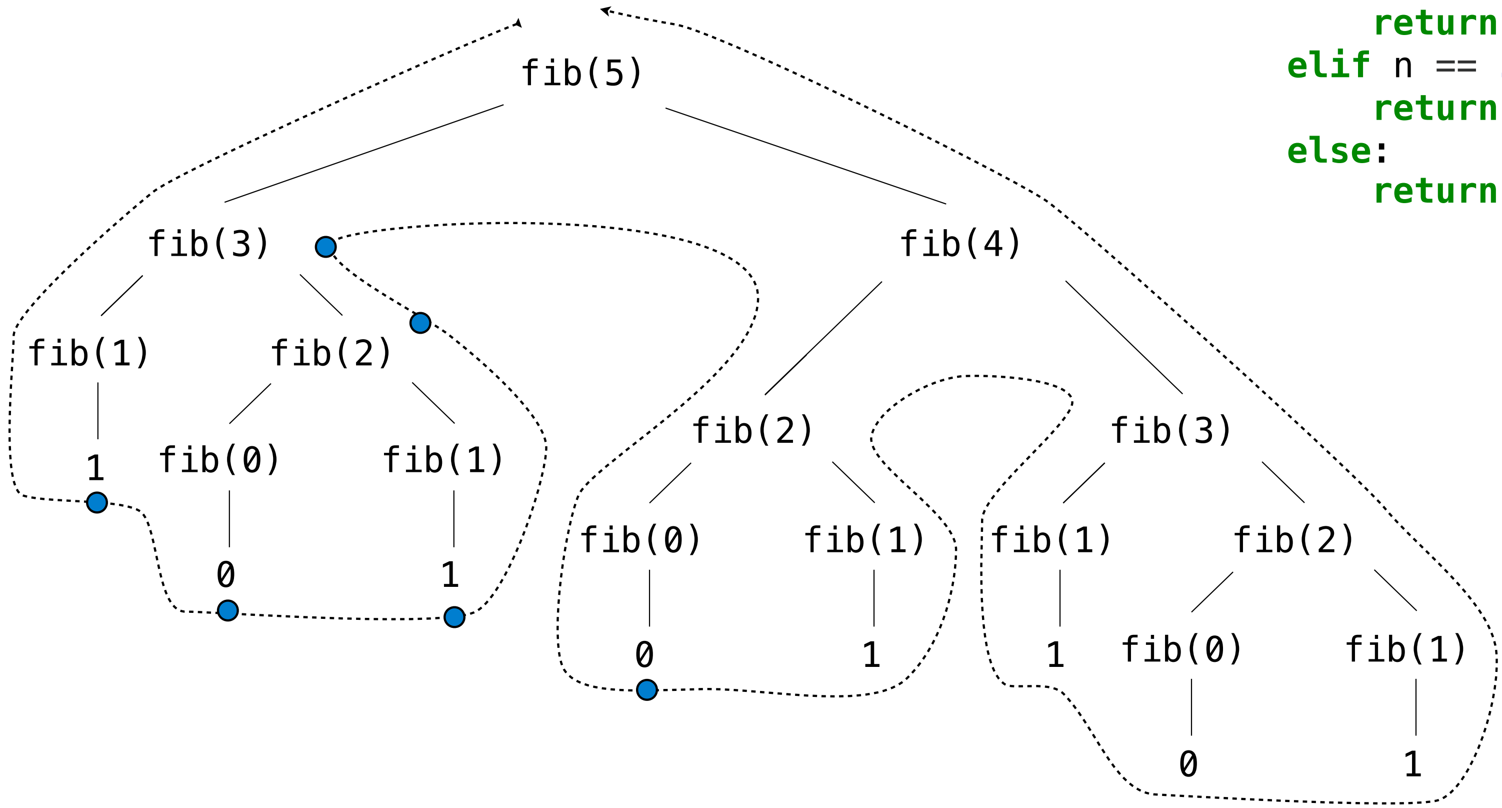

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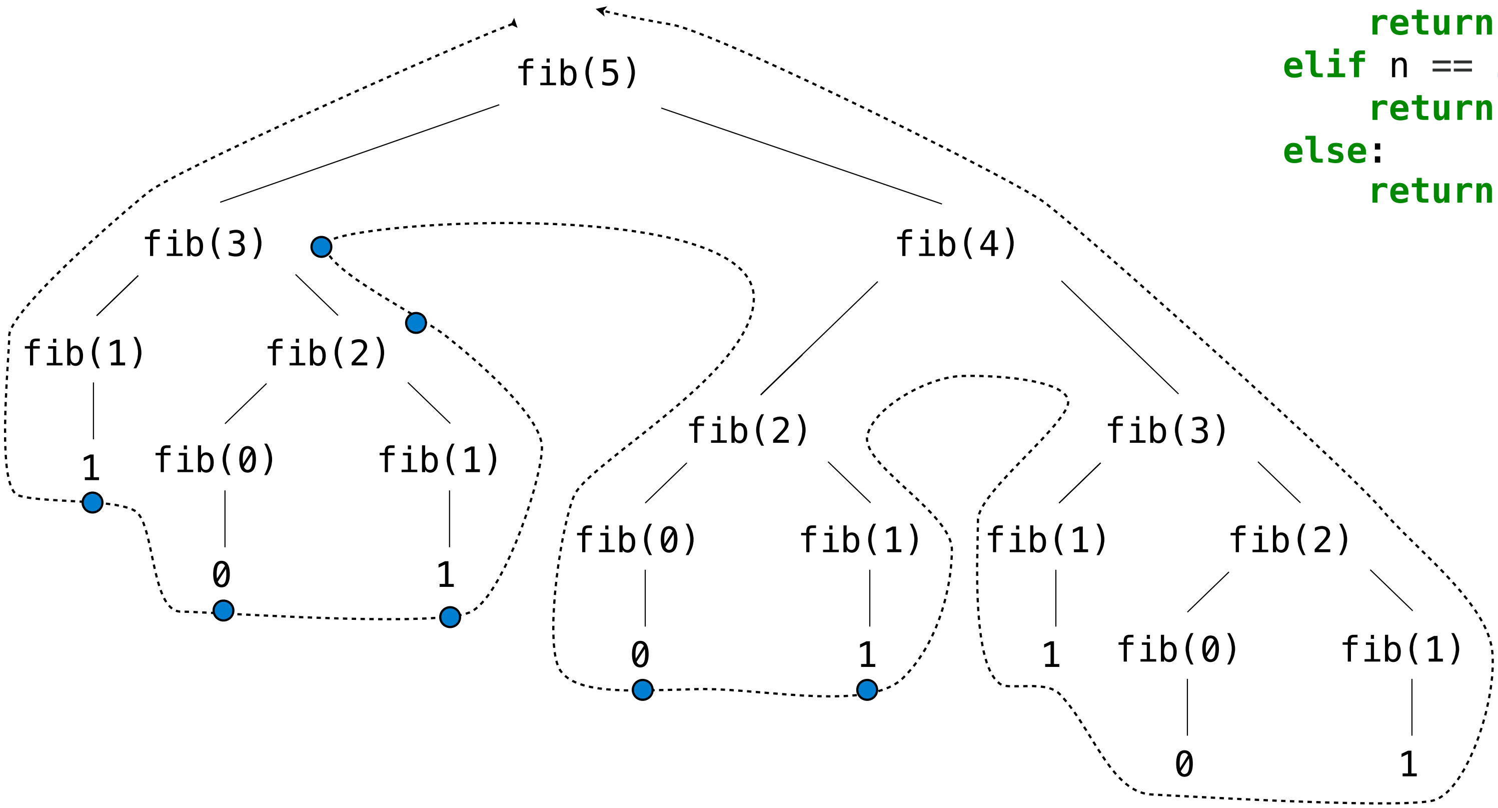


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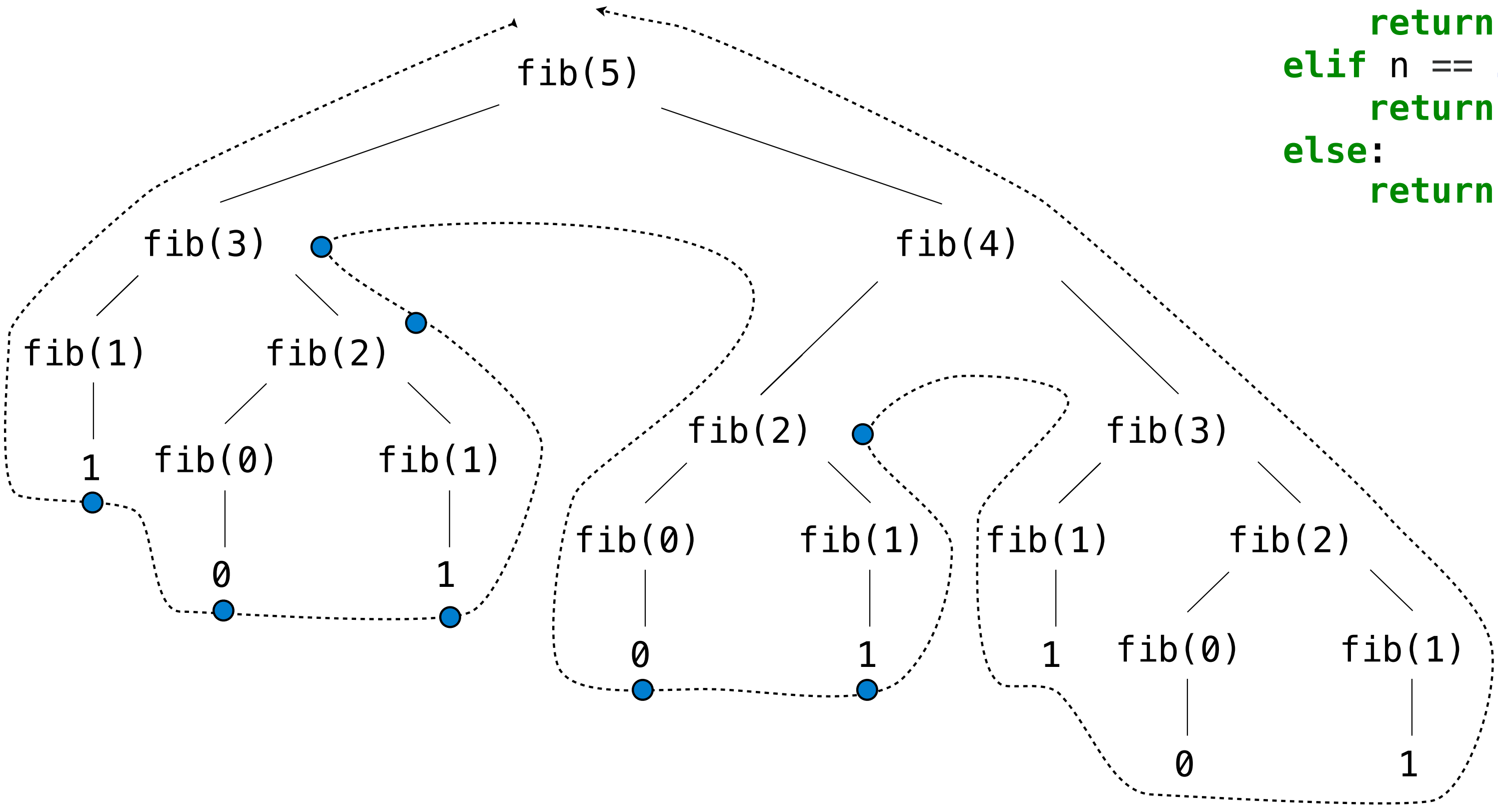

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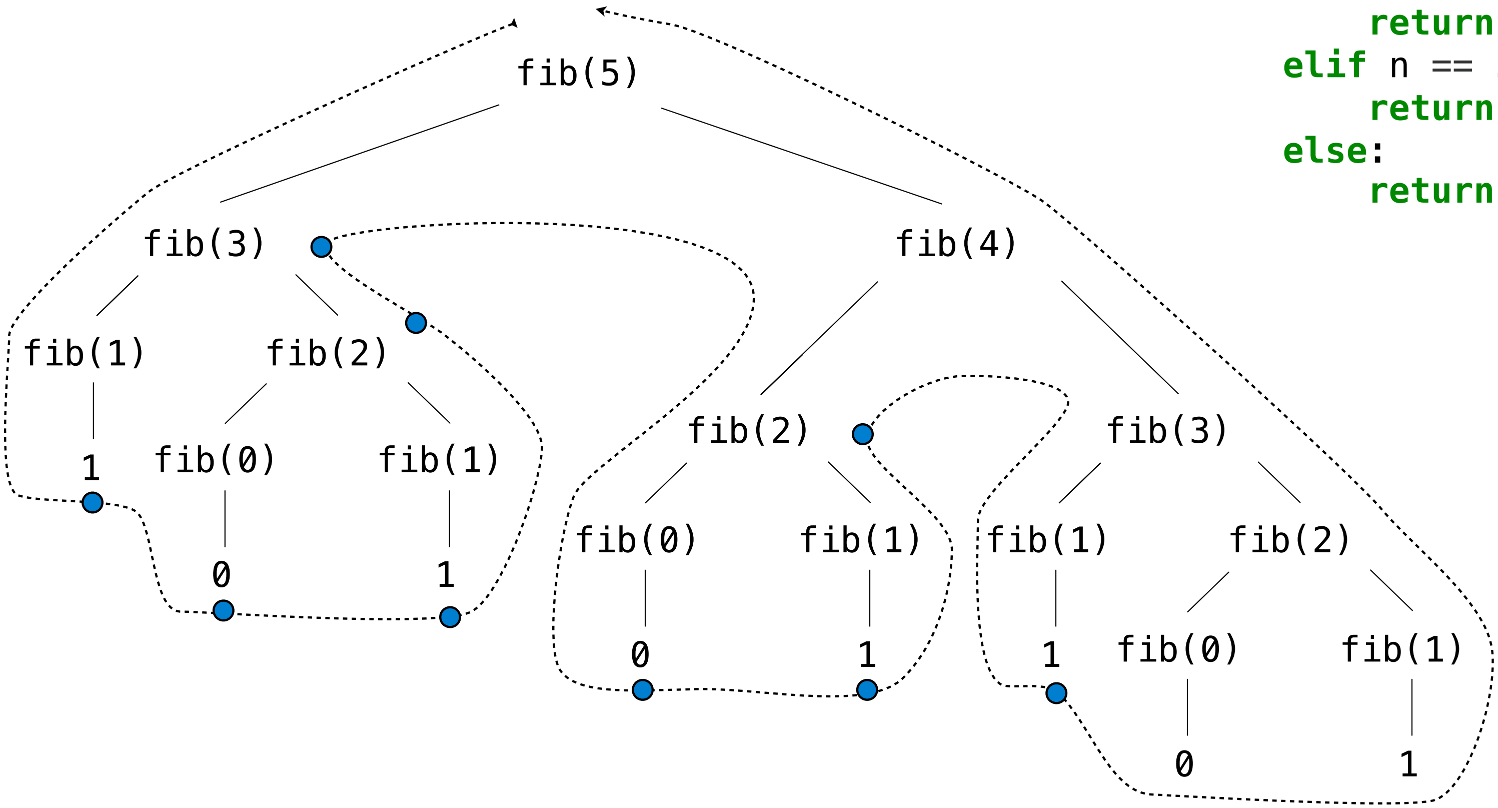

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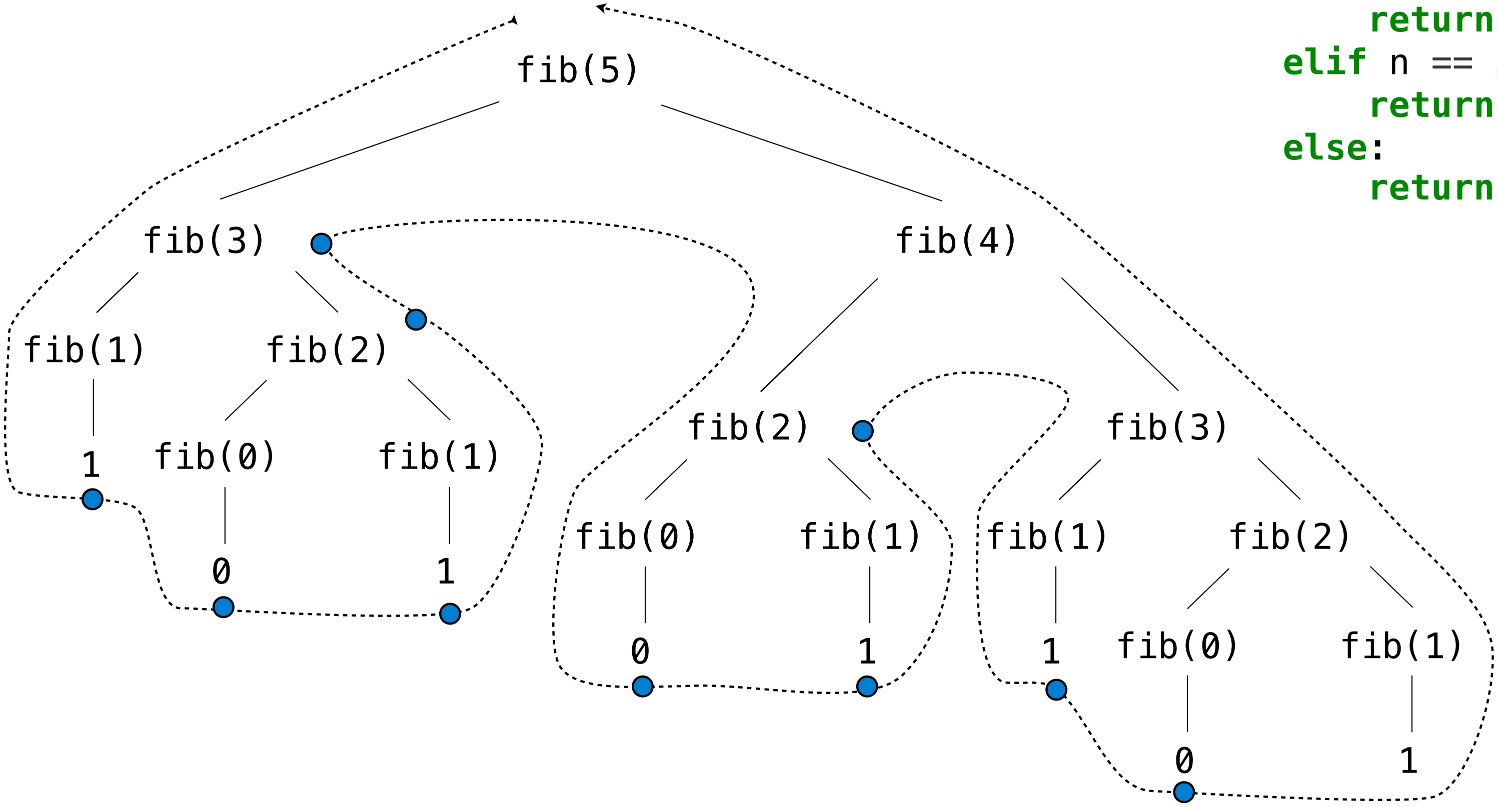

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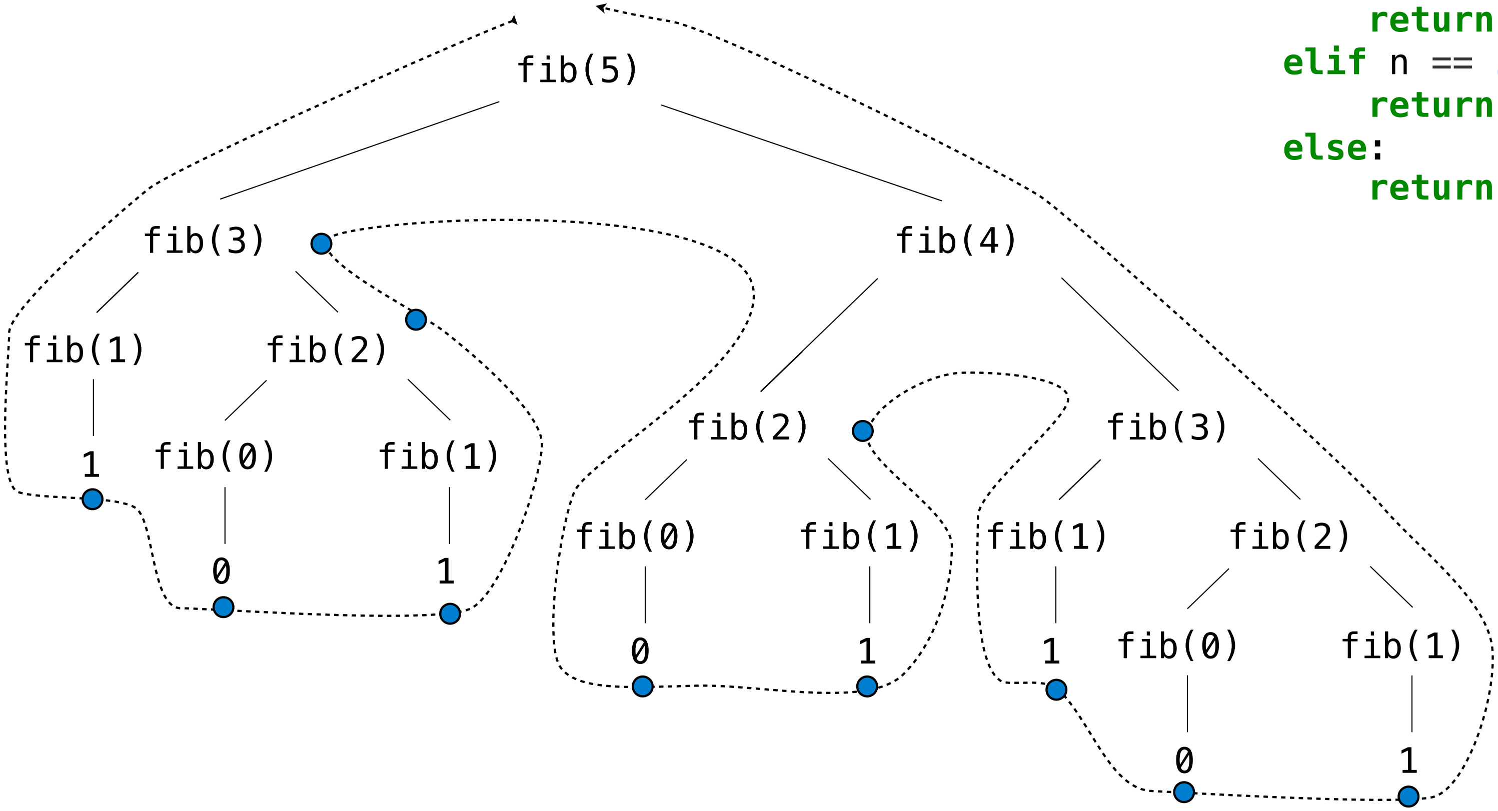

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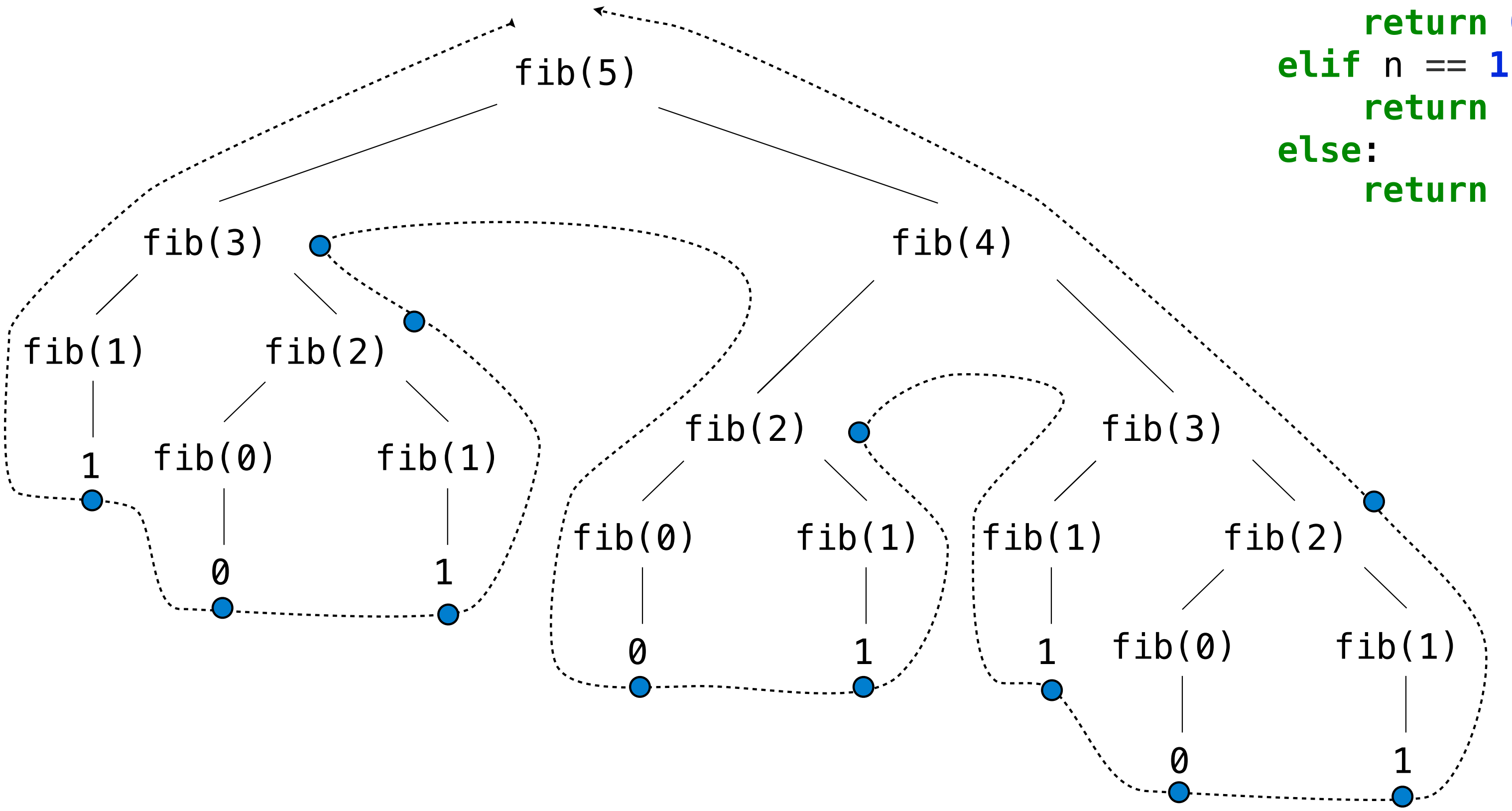


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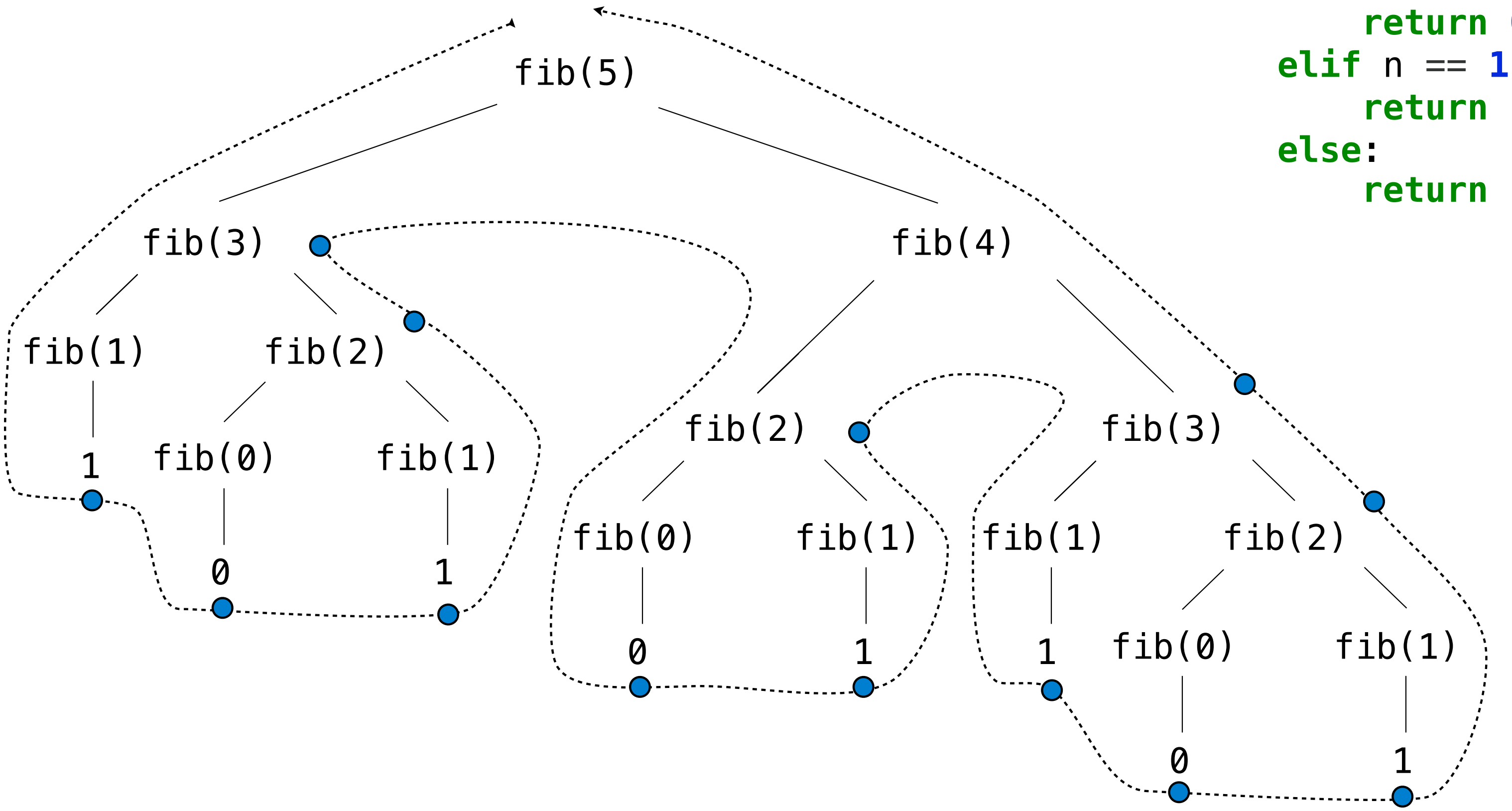

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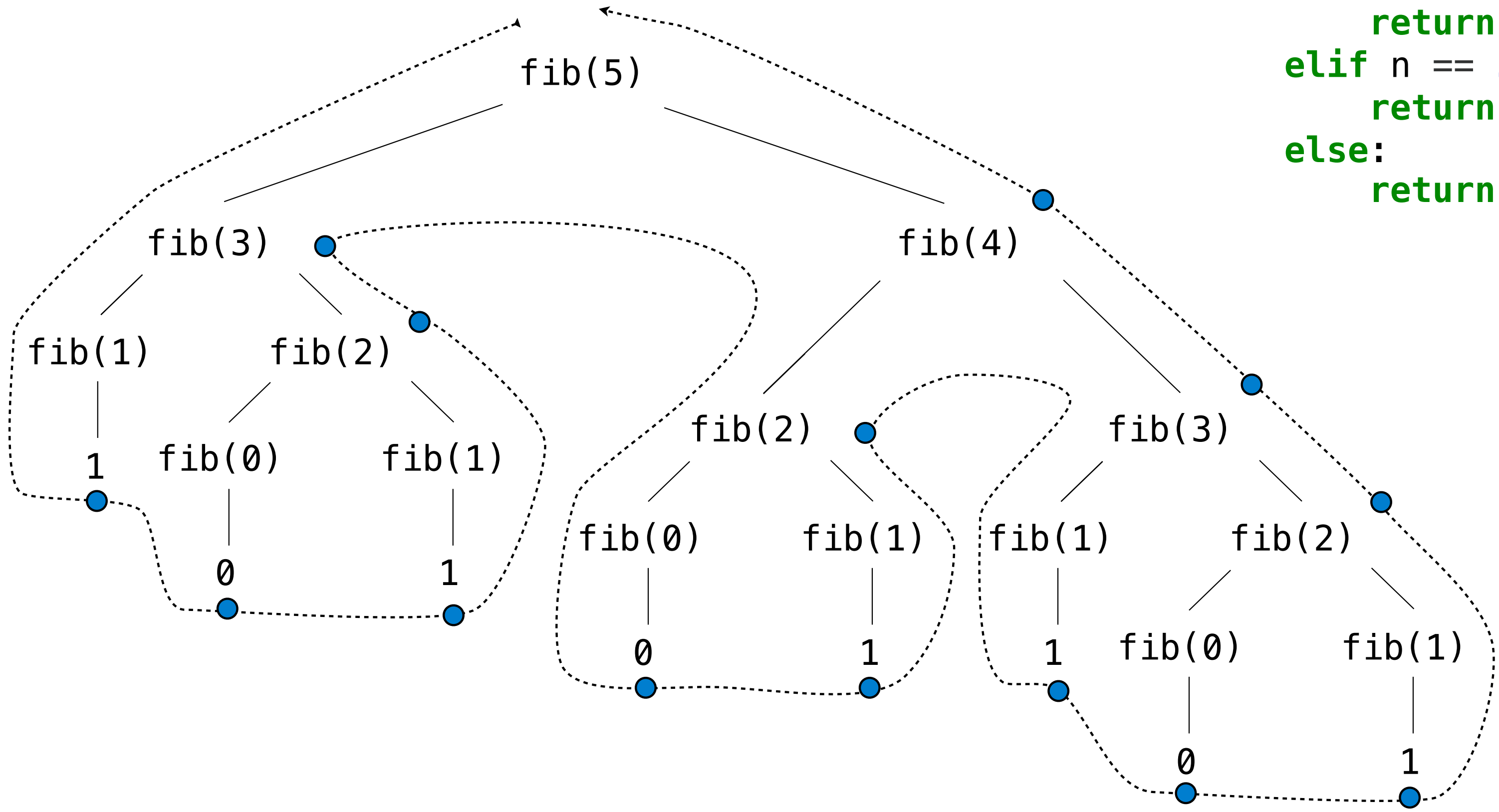
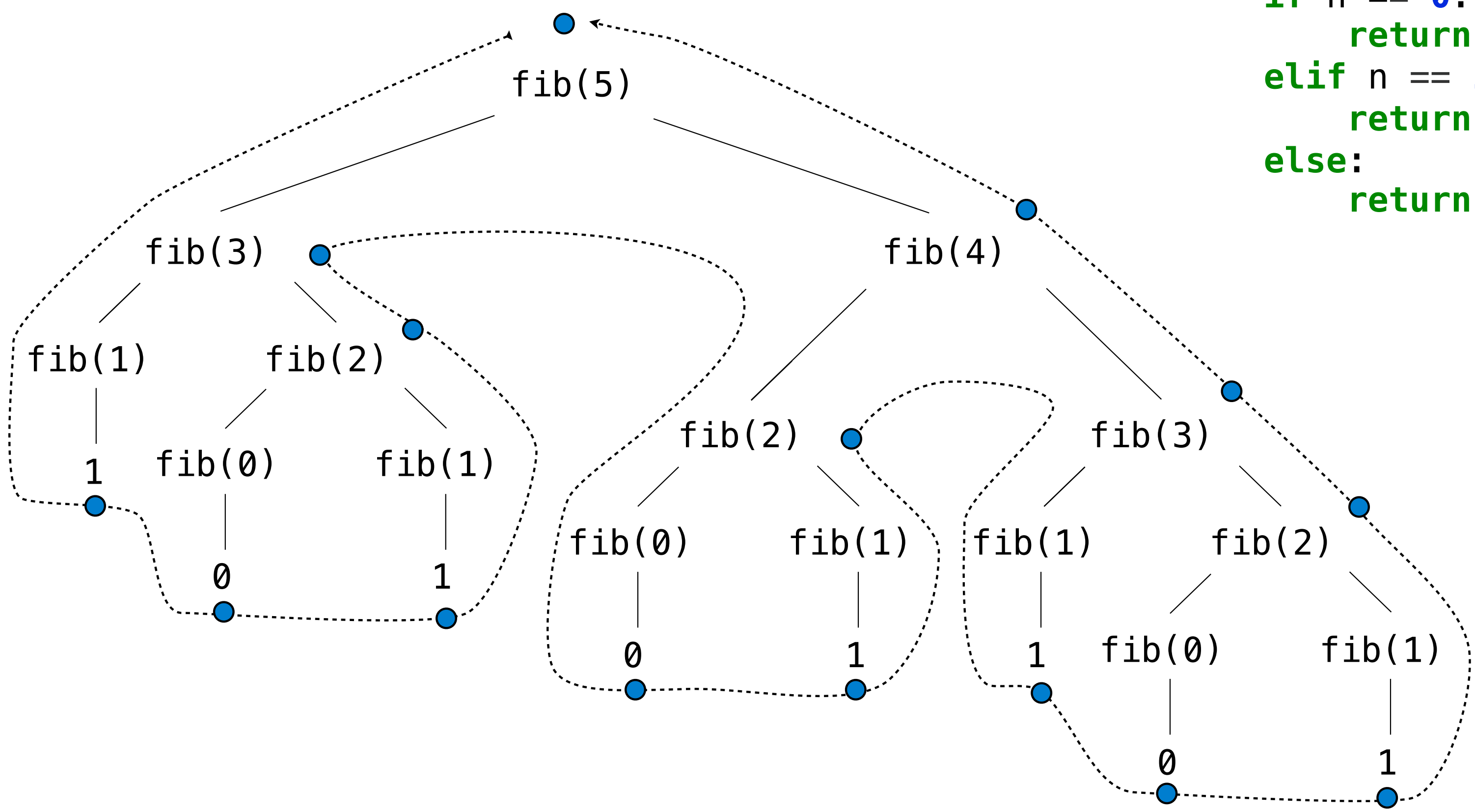


http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

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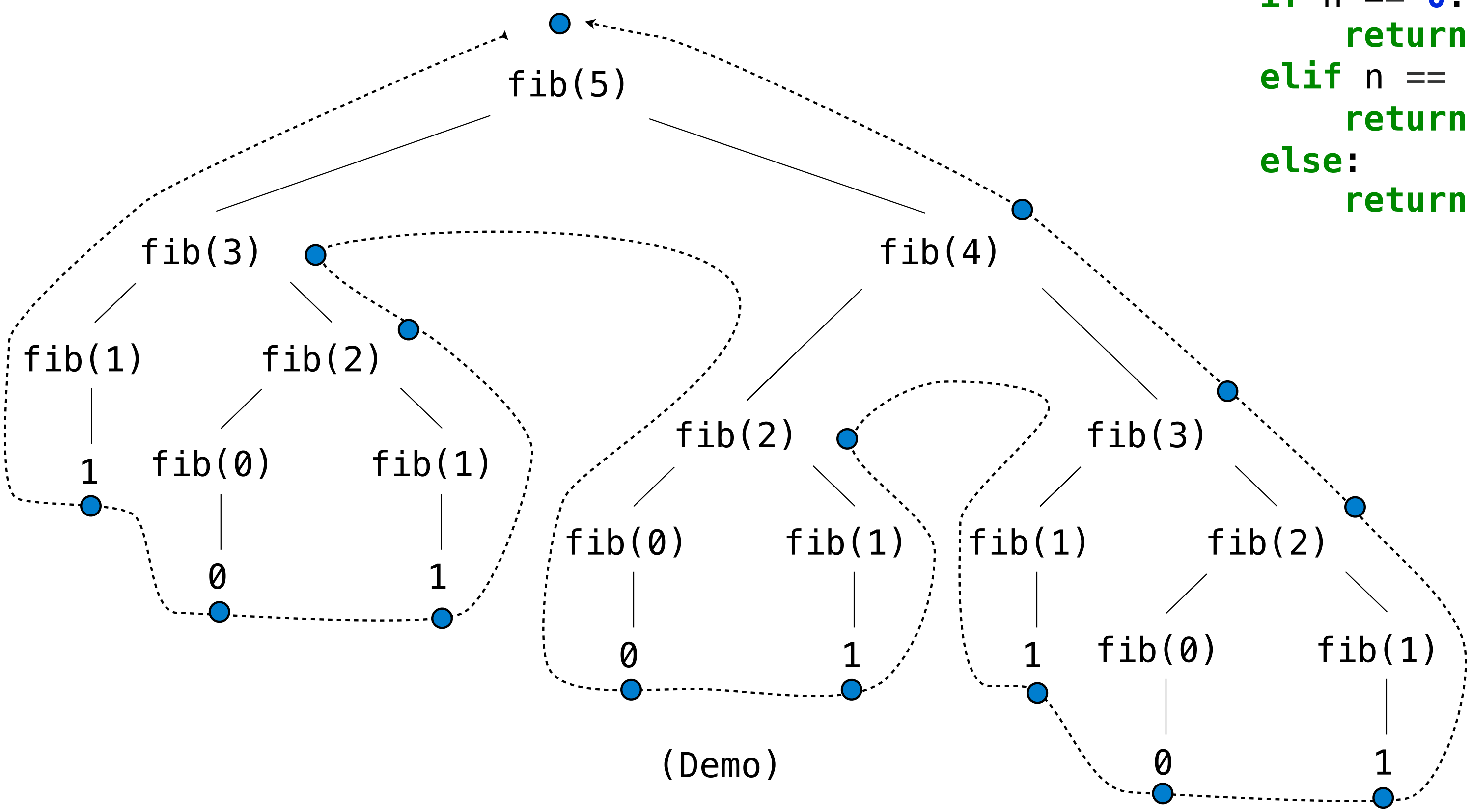

(Demo)

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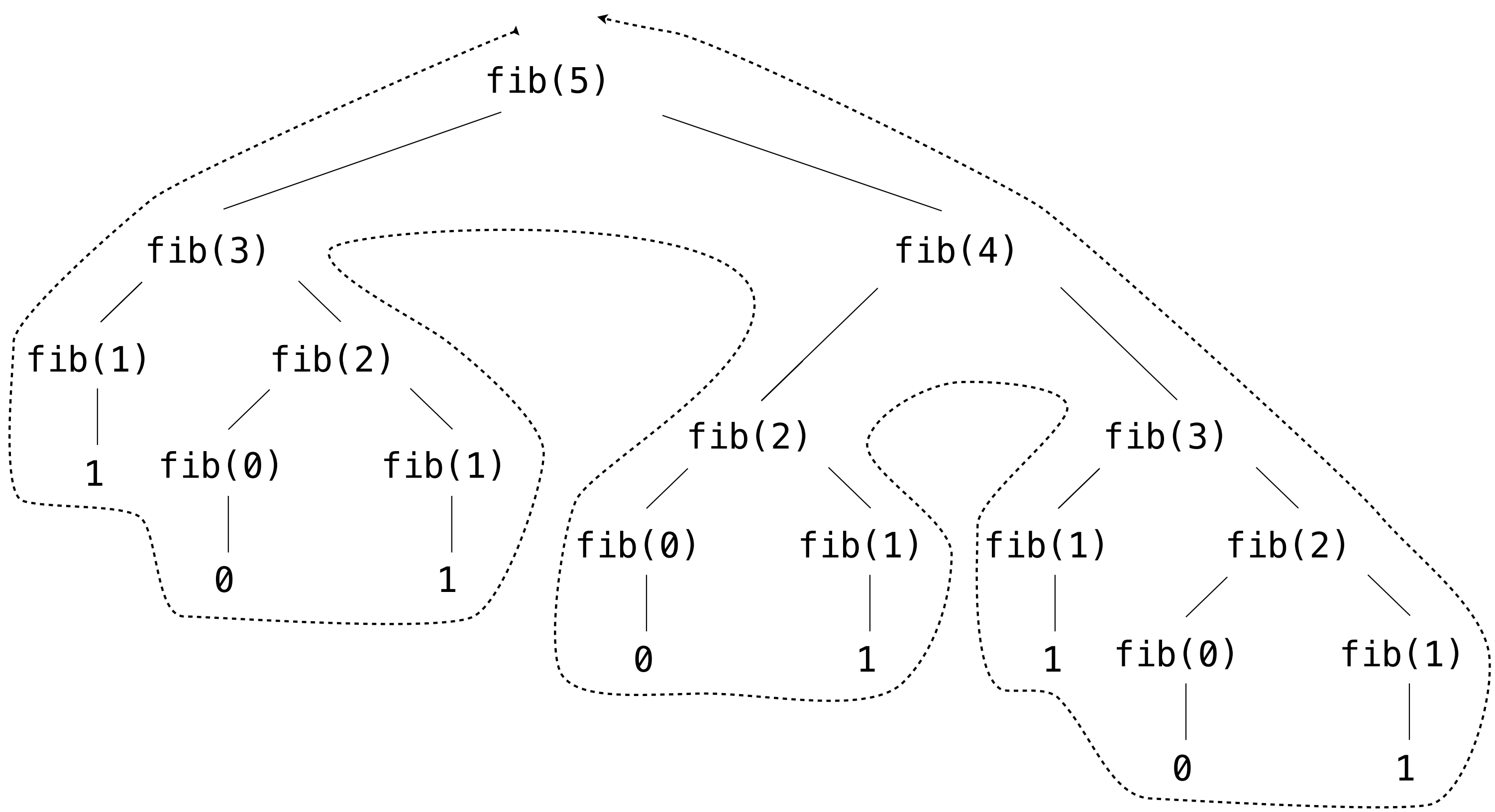

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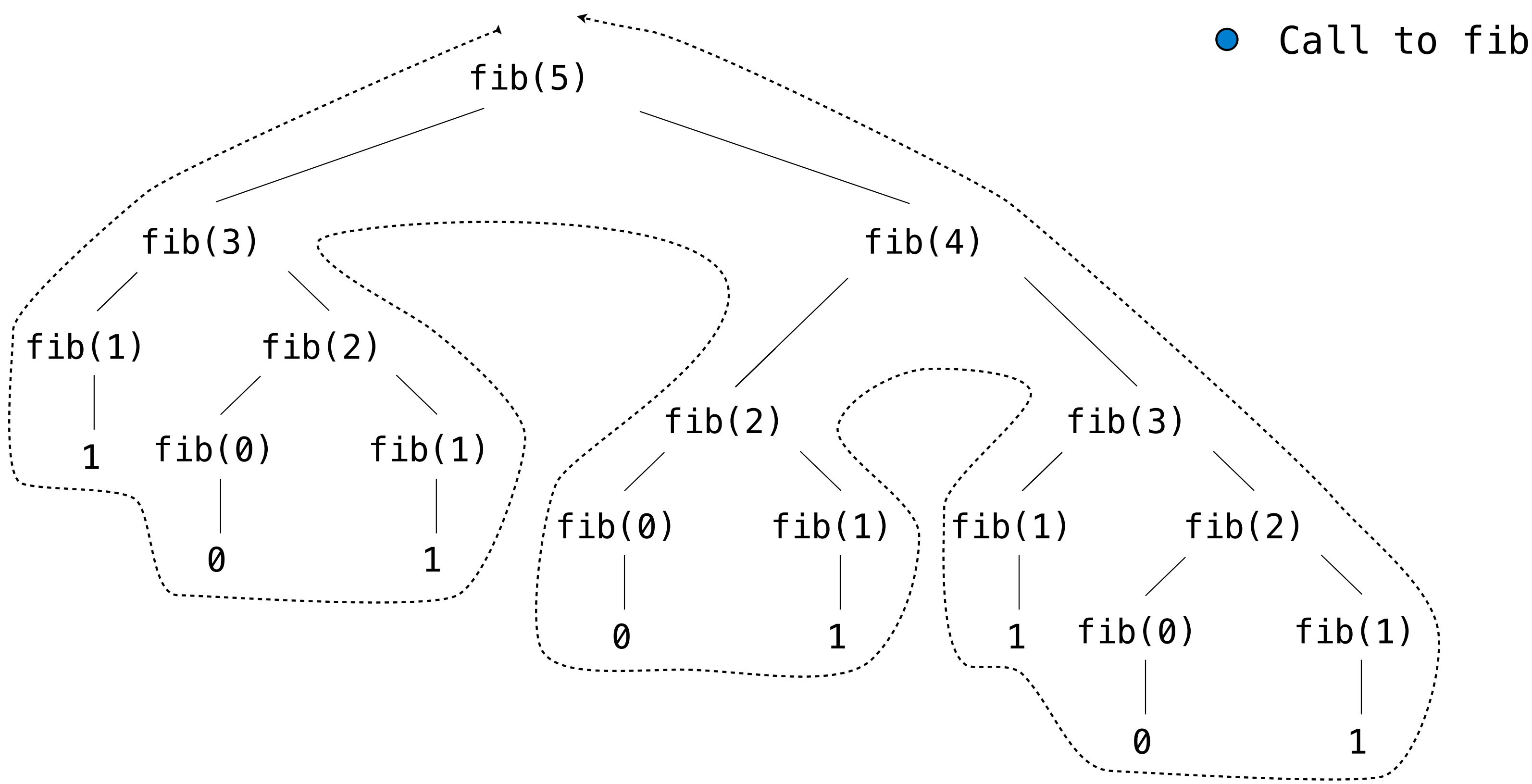

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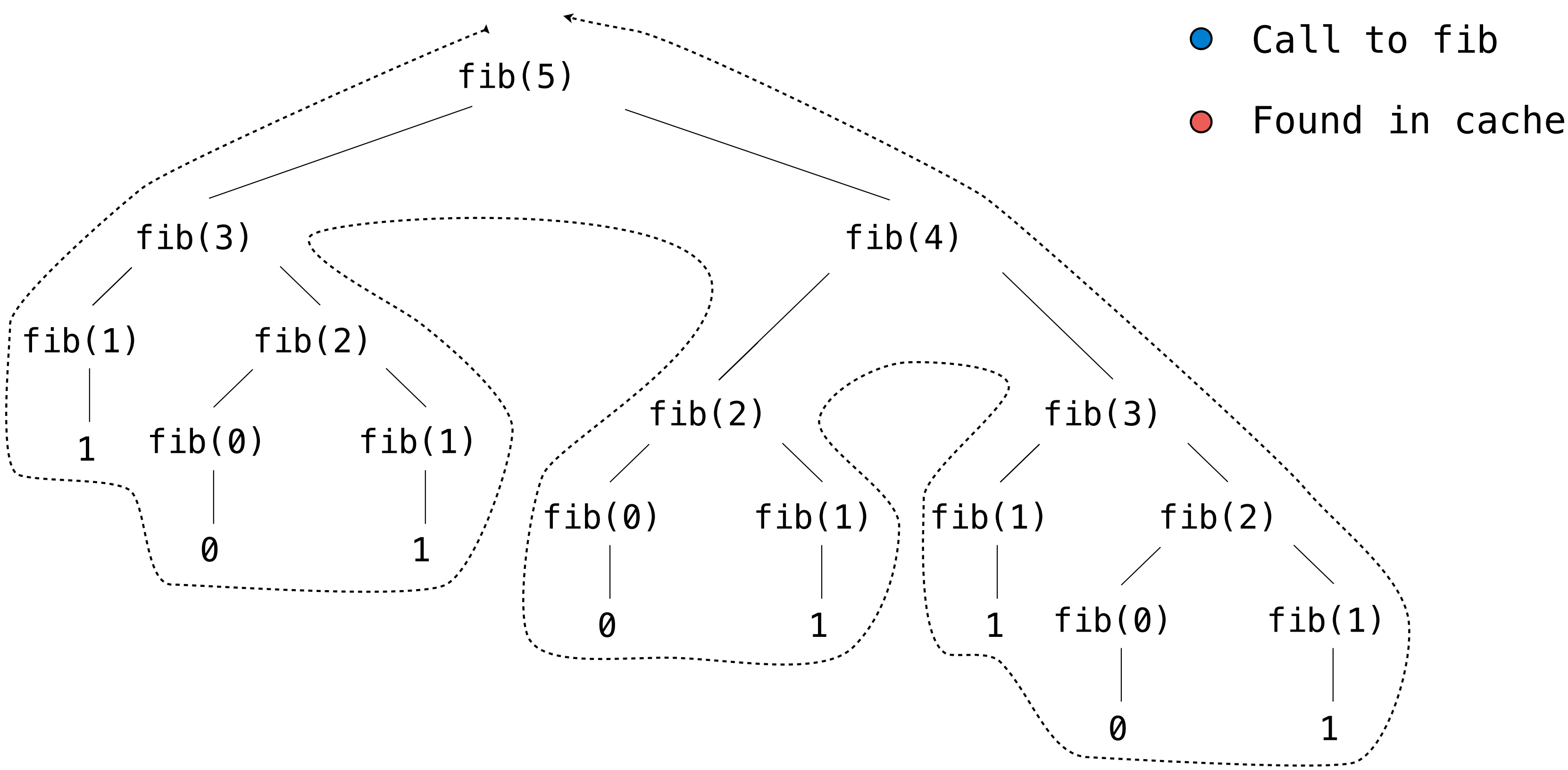

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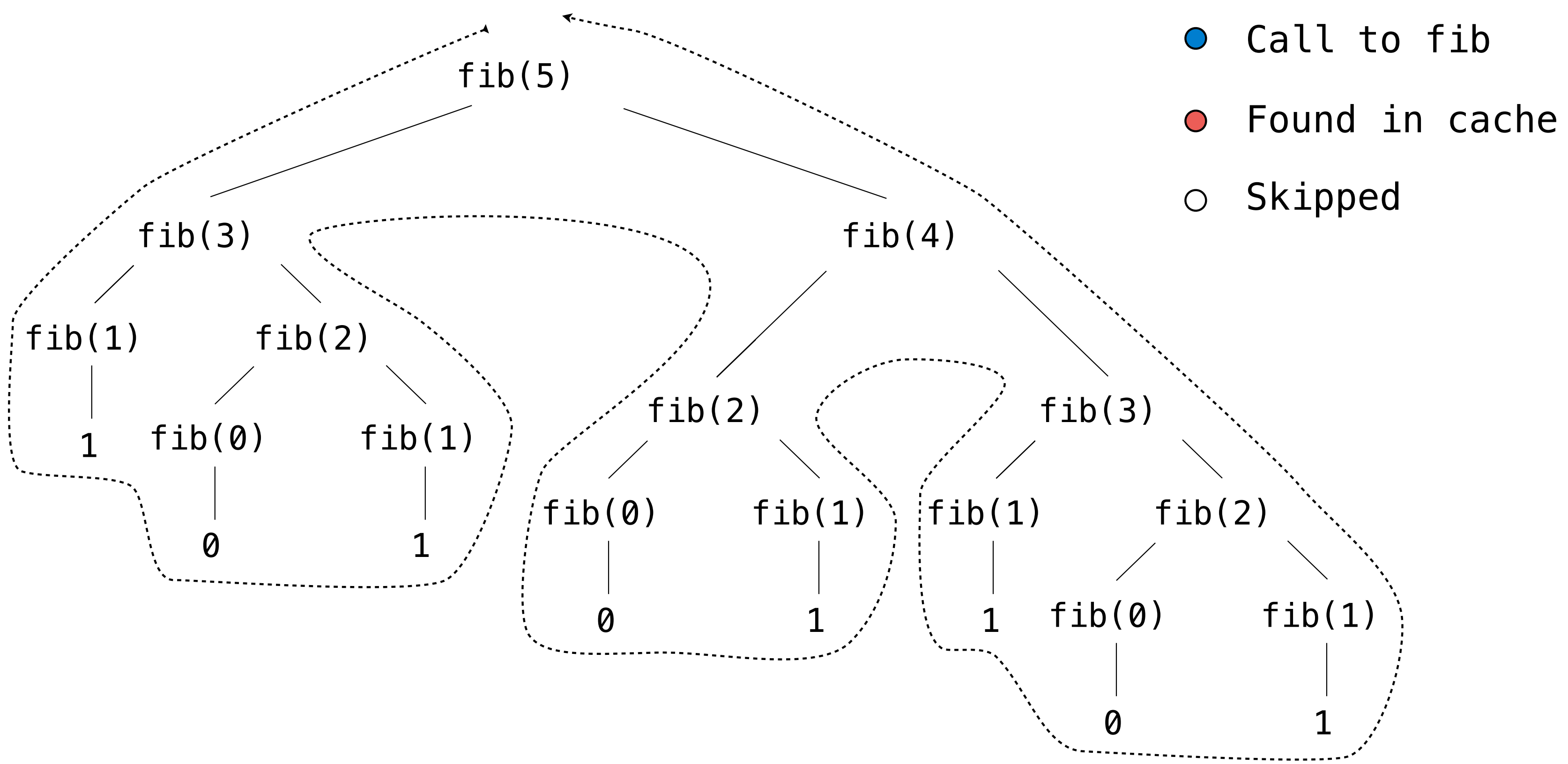

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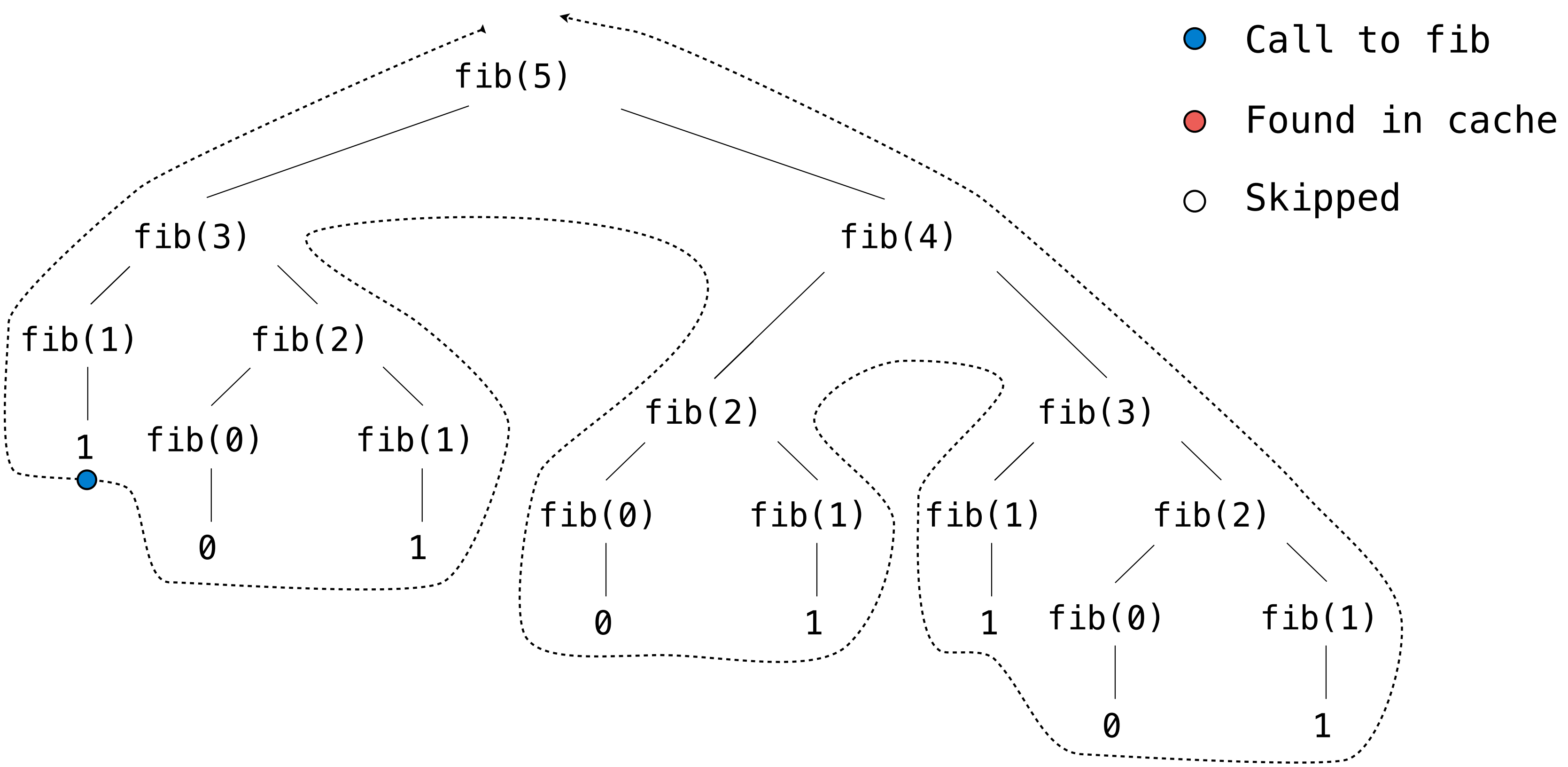

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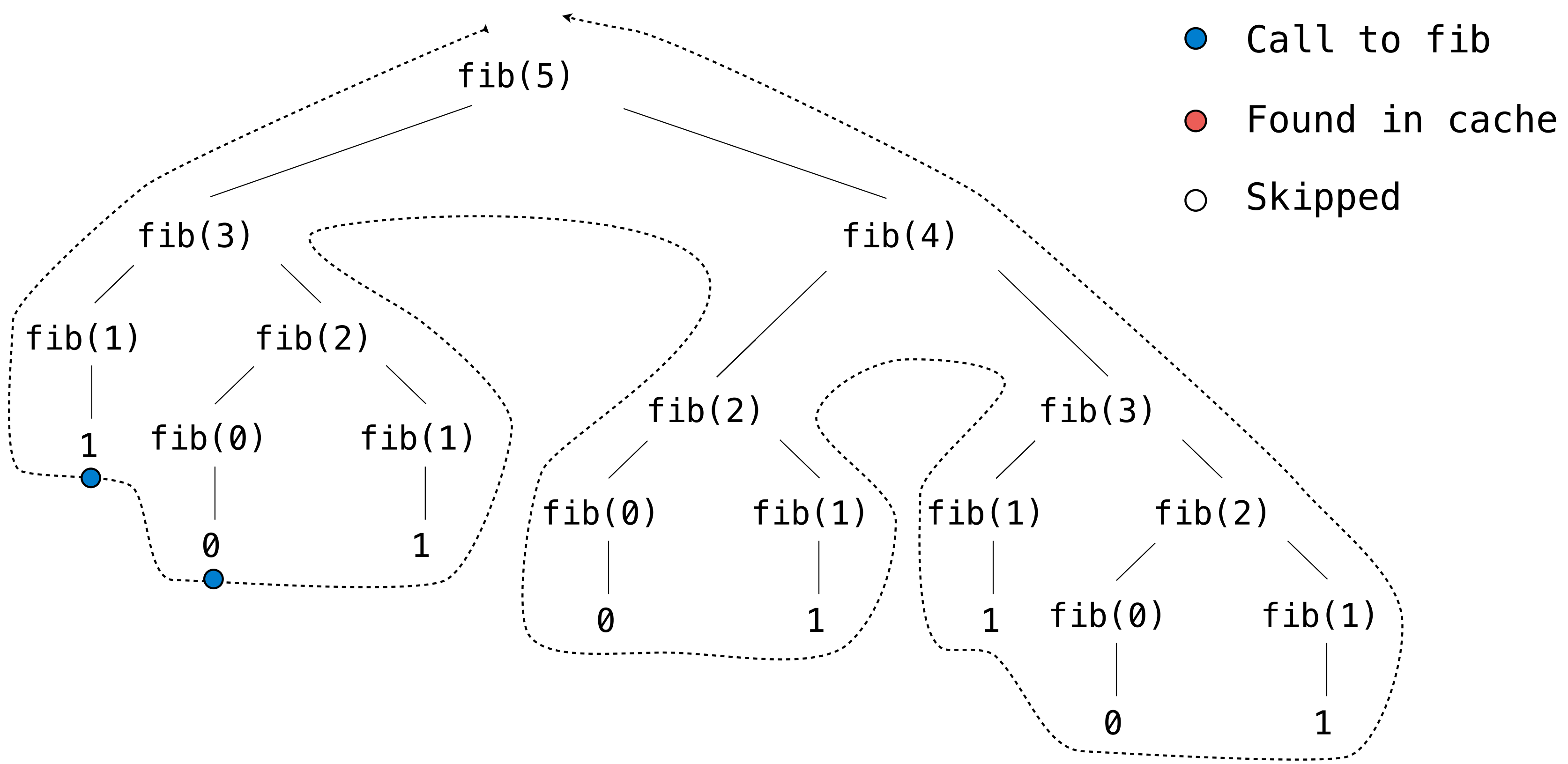

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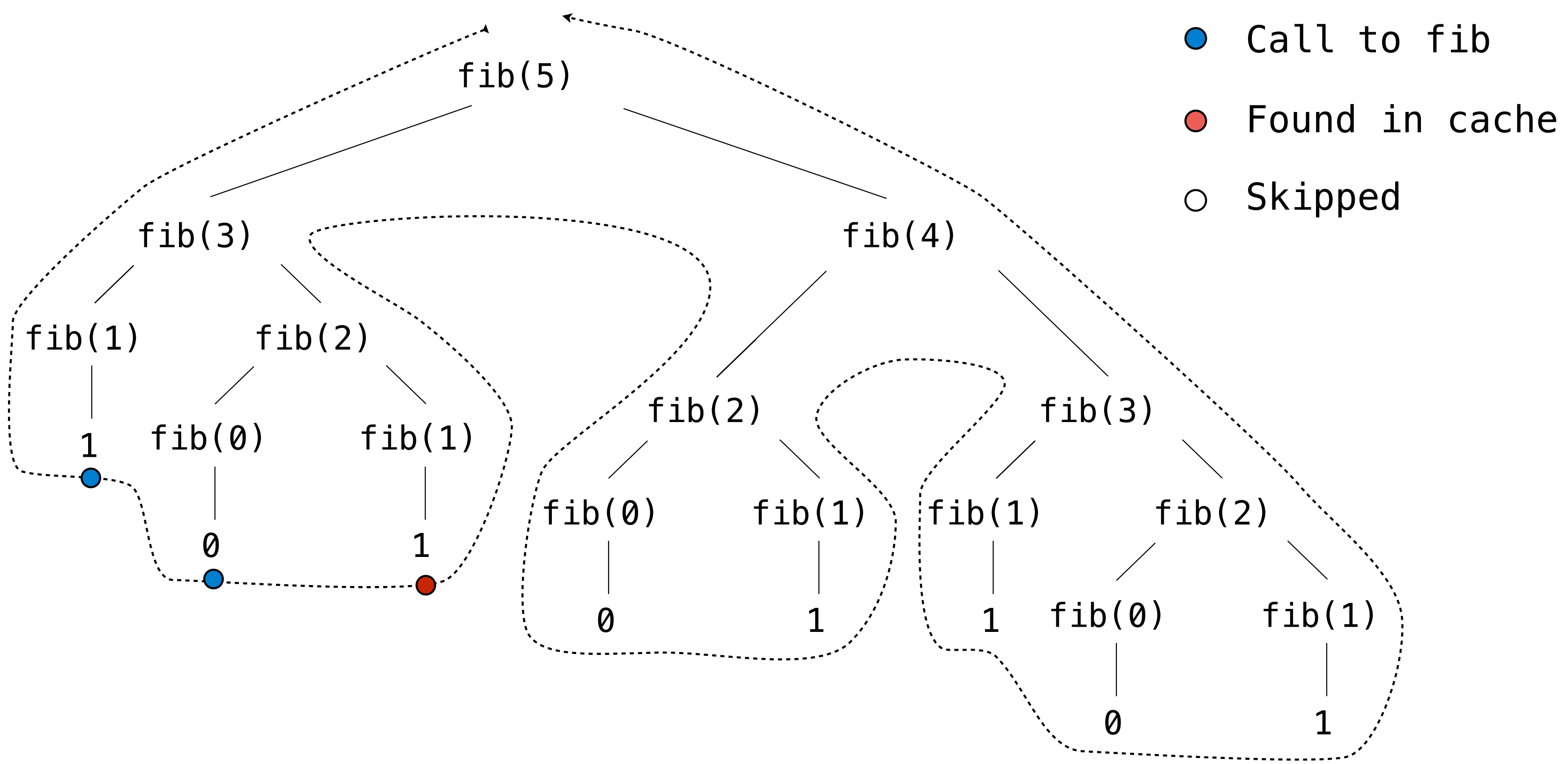

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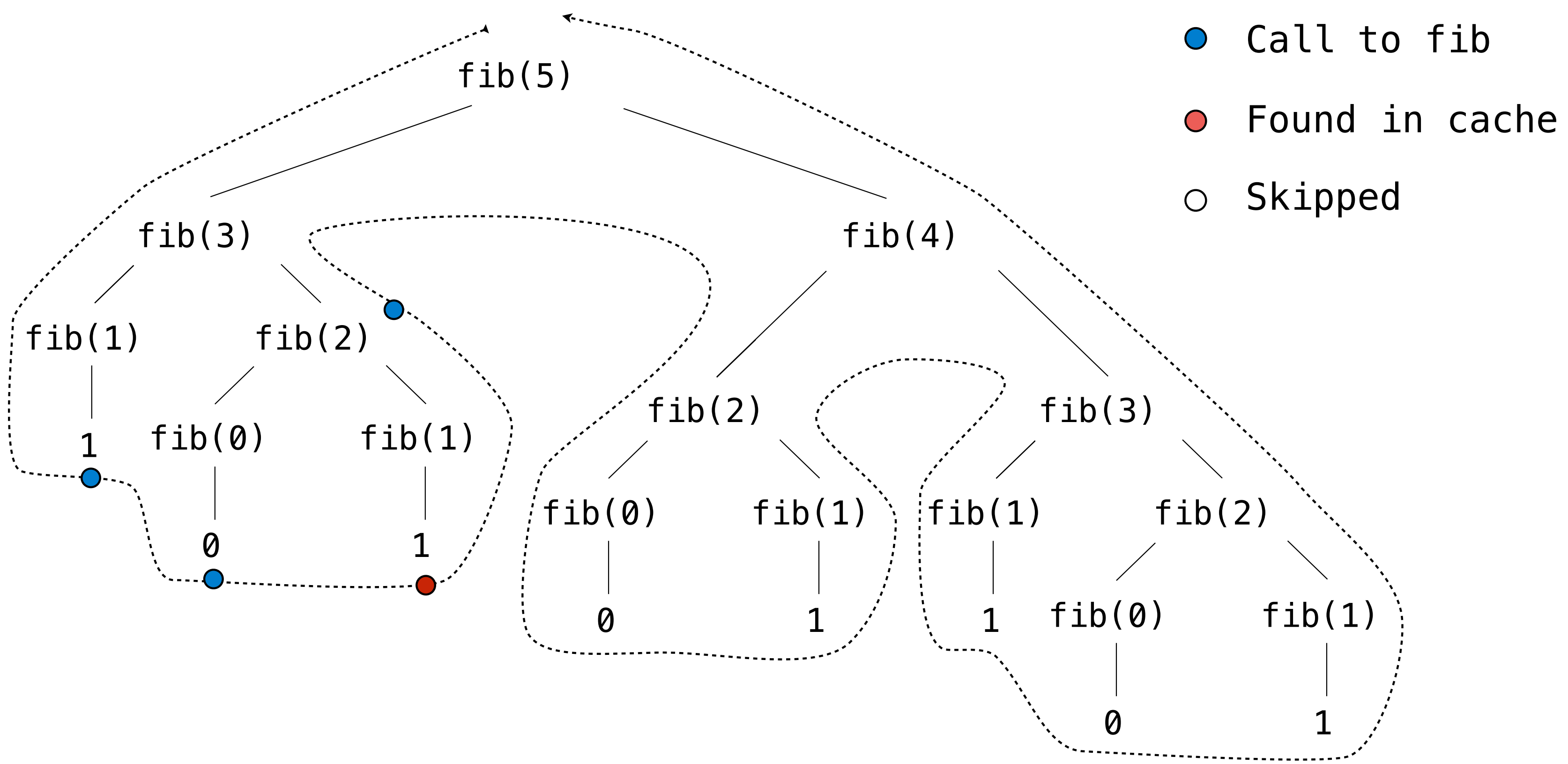

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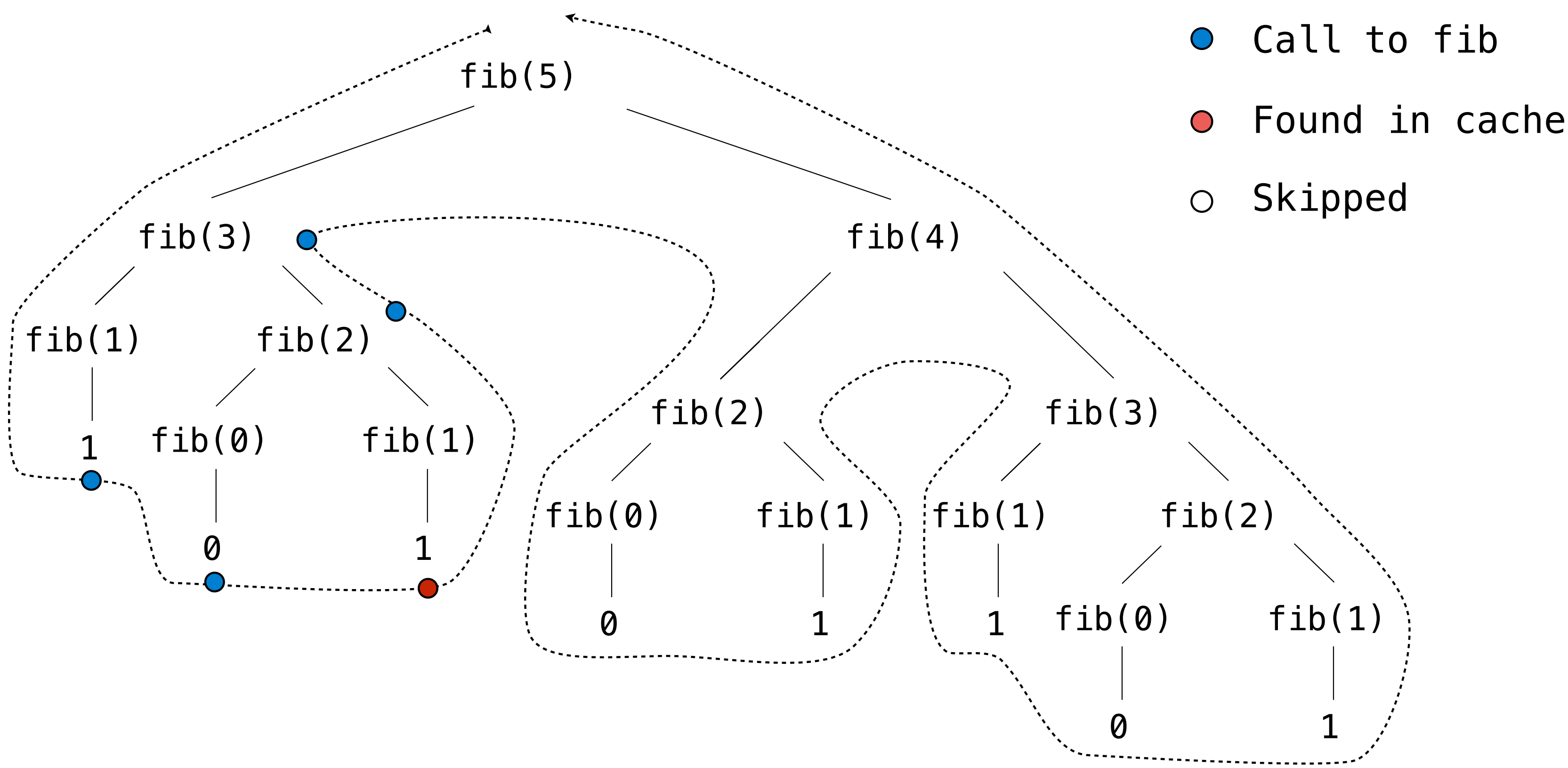

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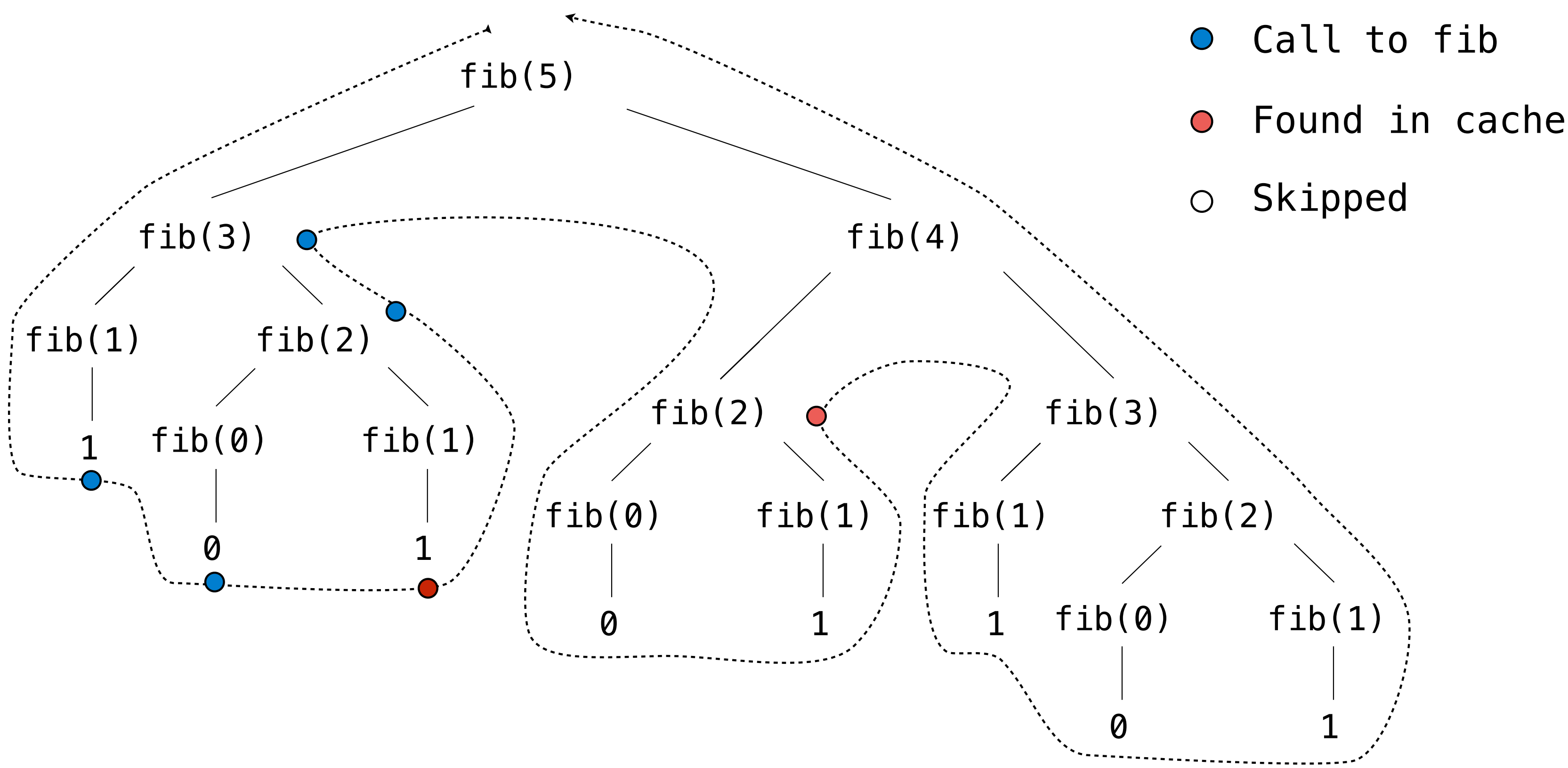

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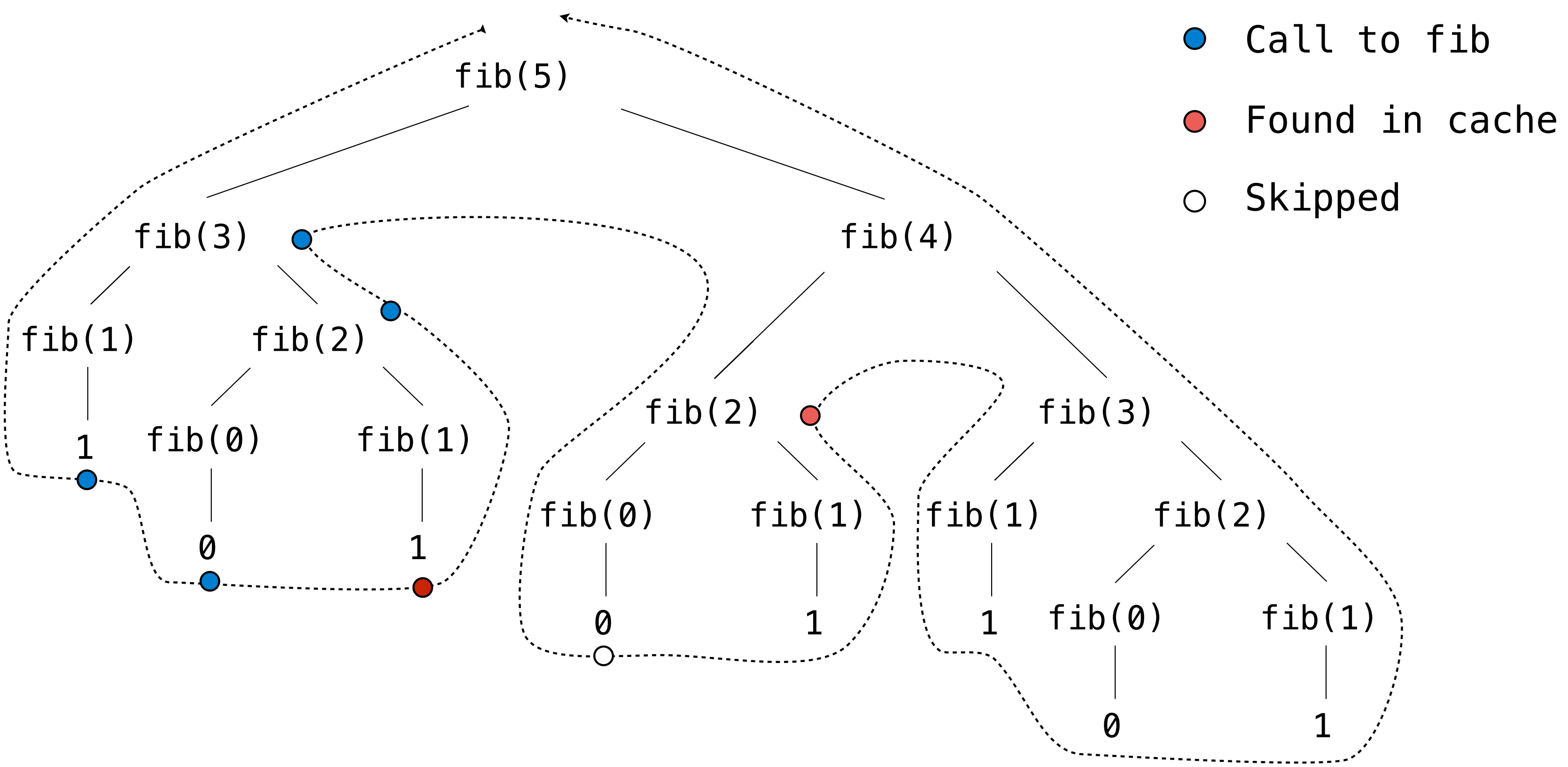

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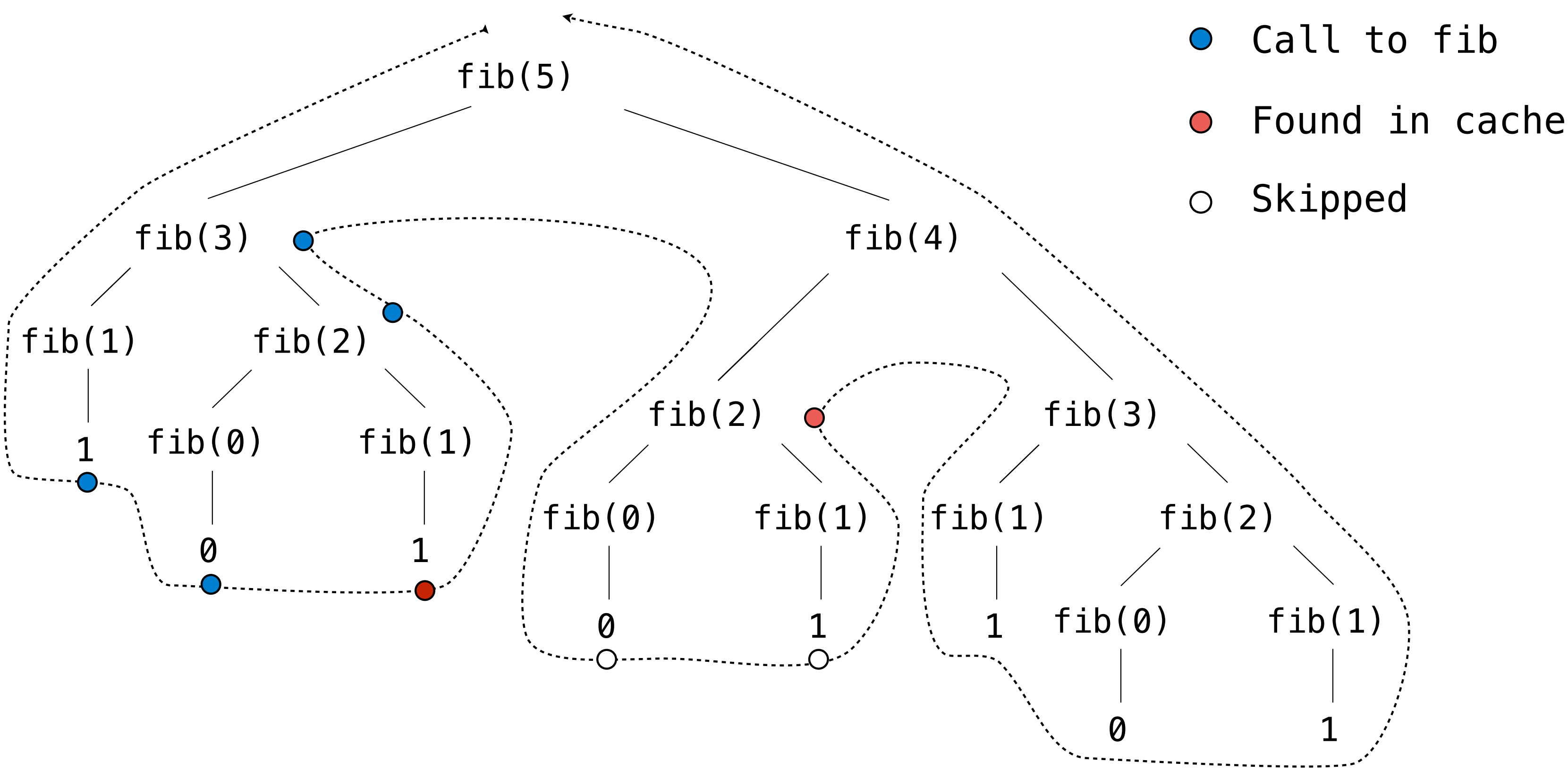

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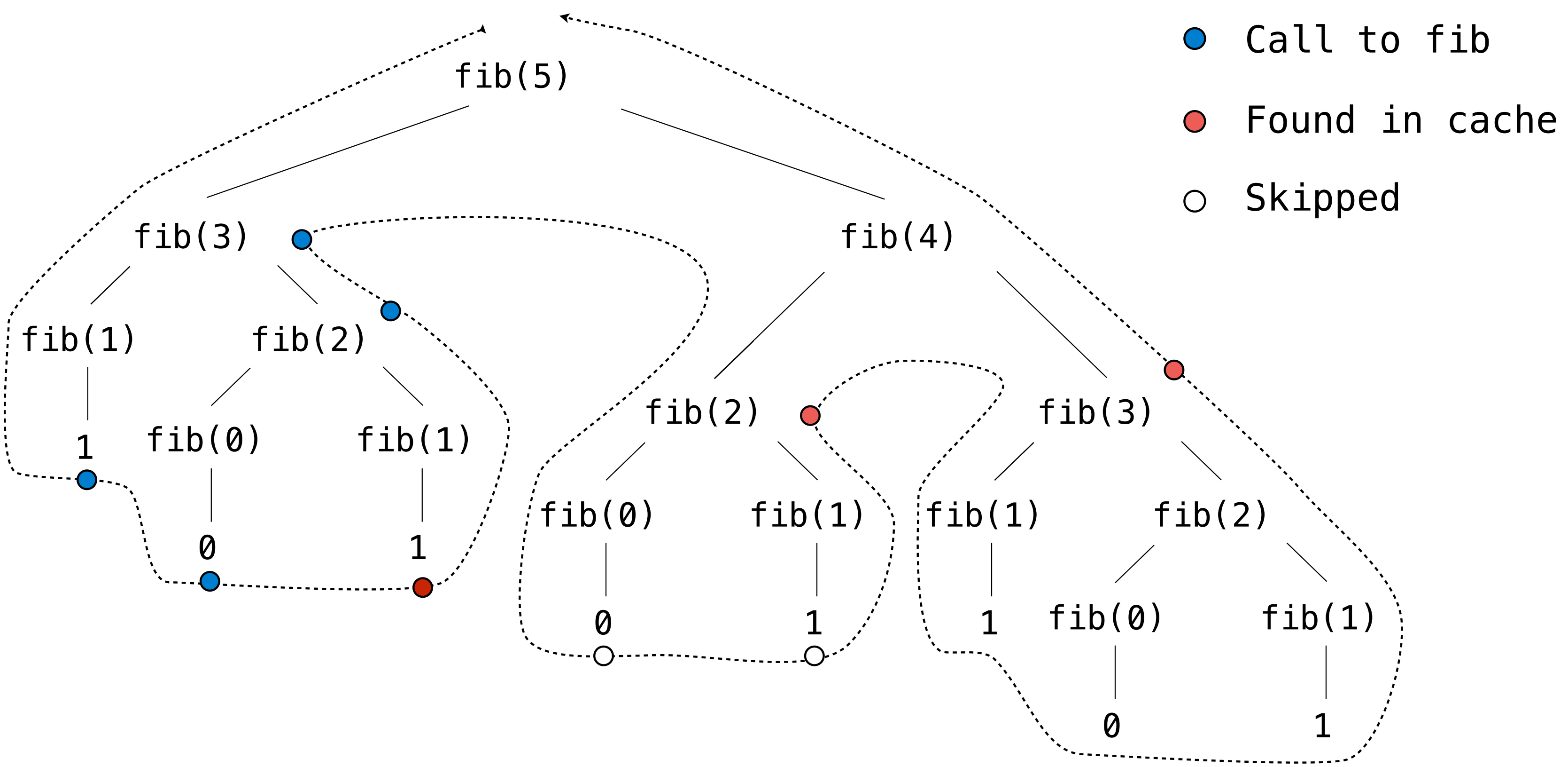

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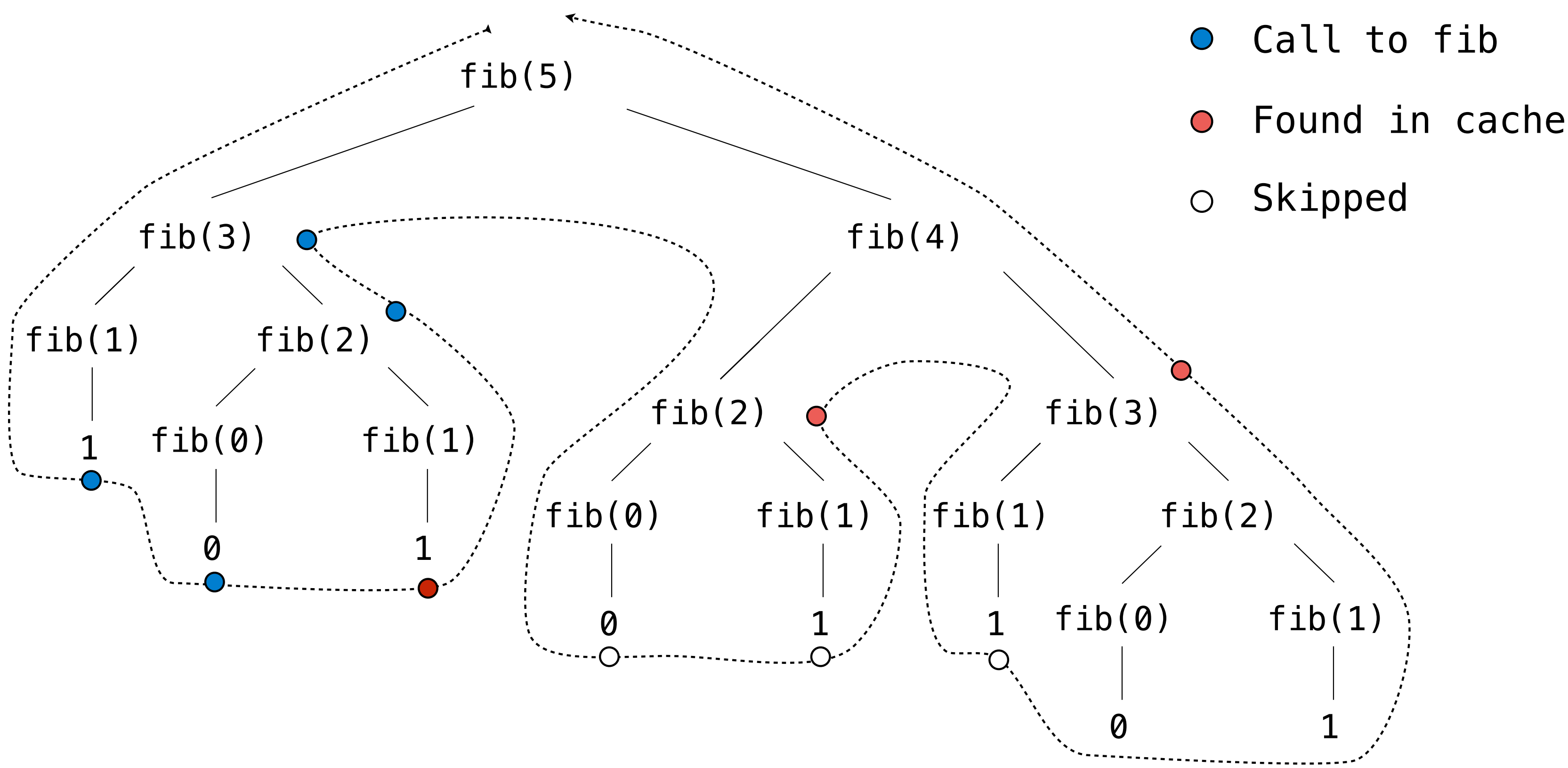

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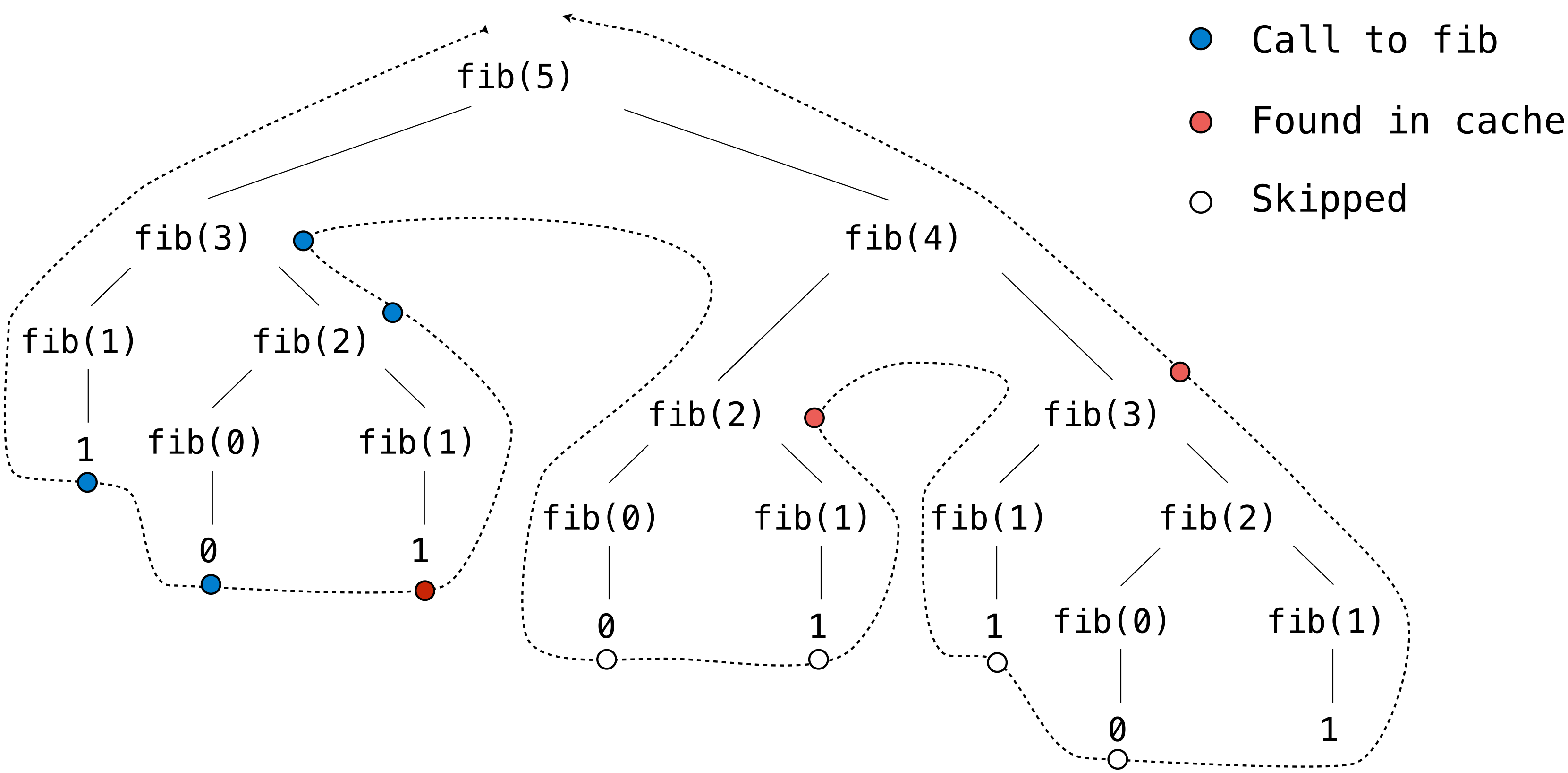

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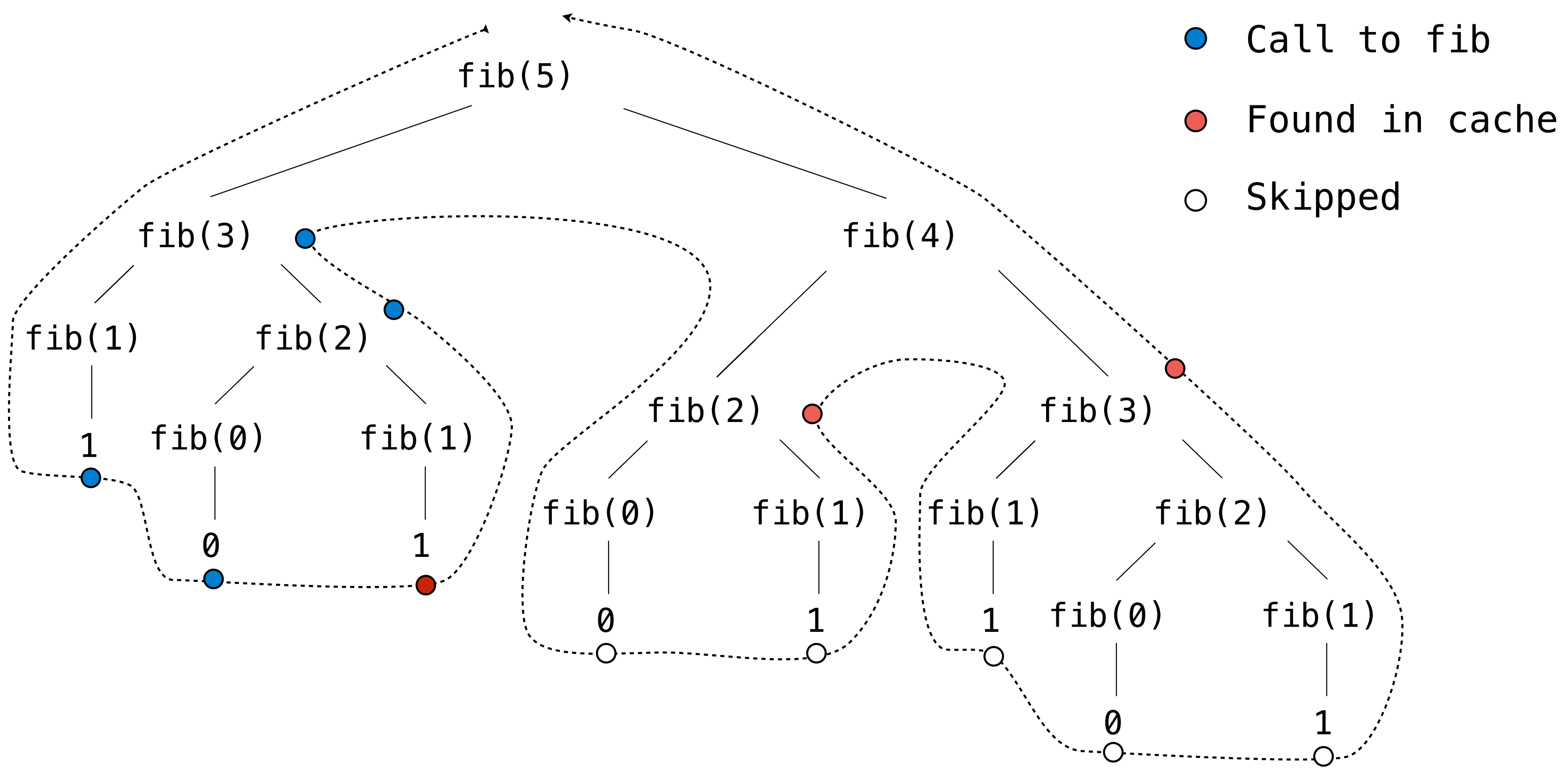

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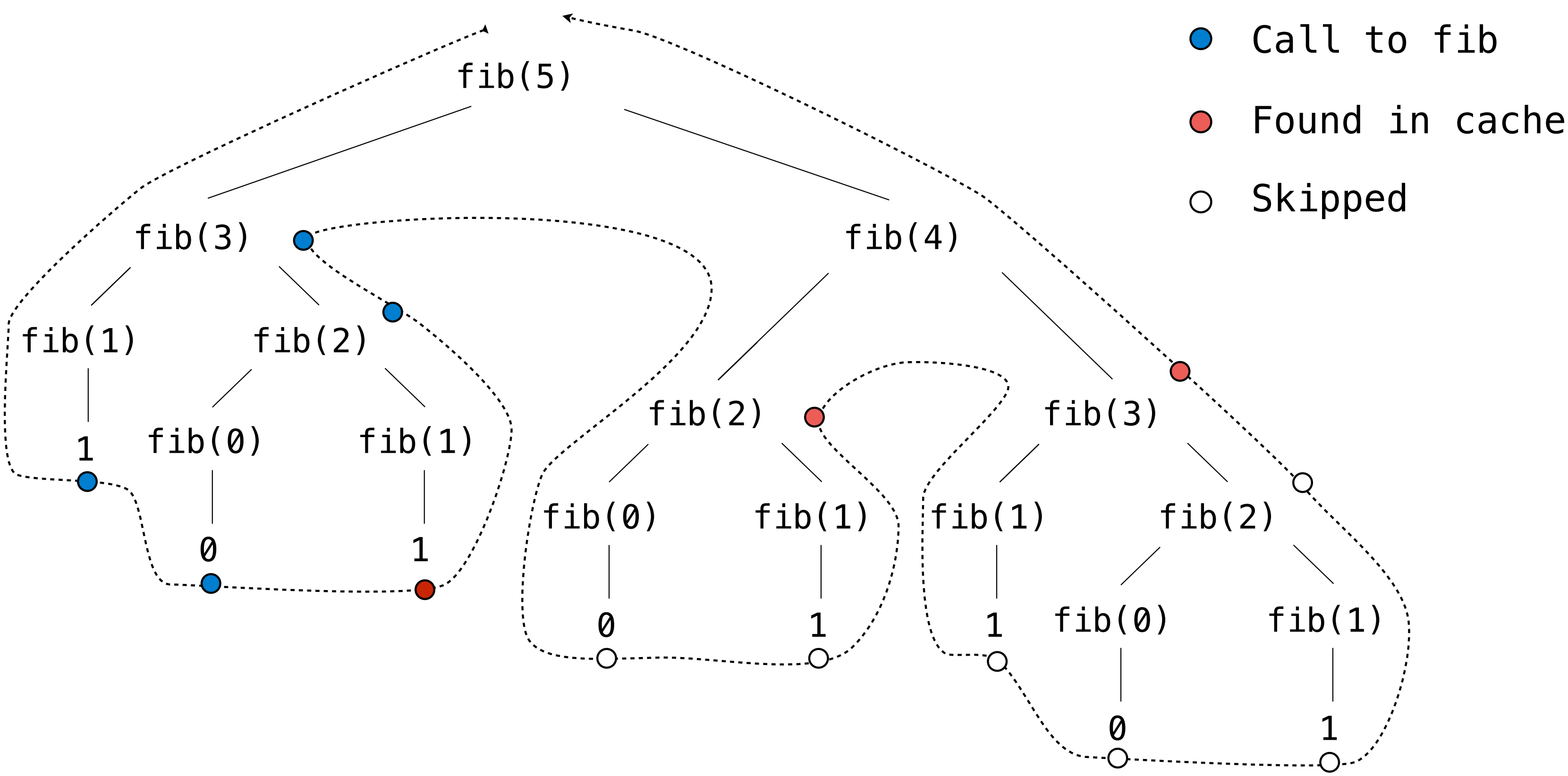

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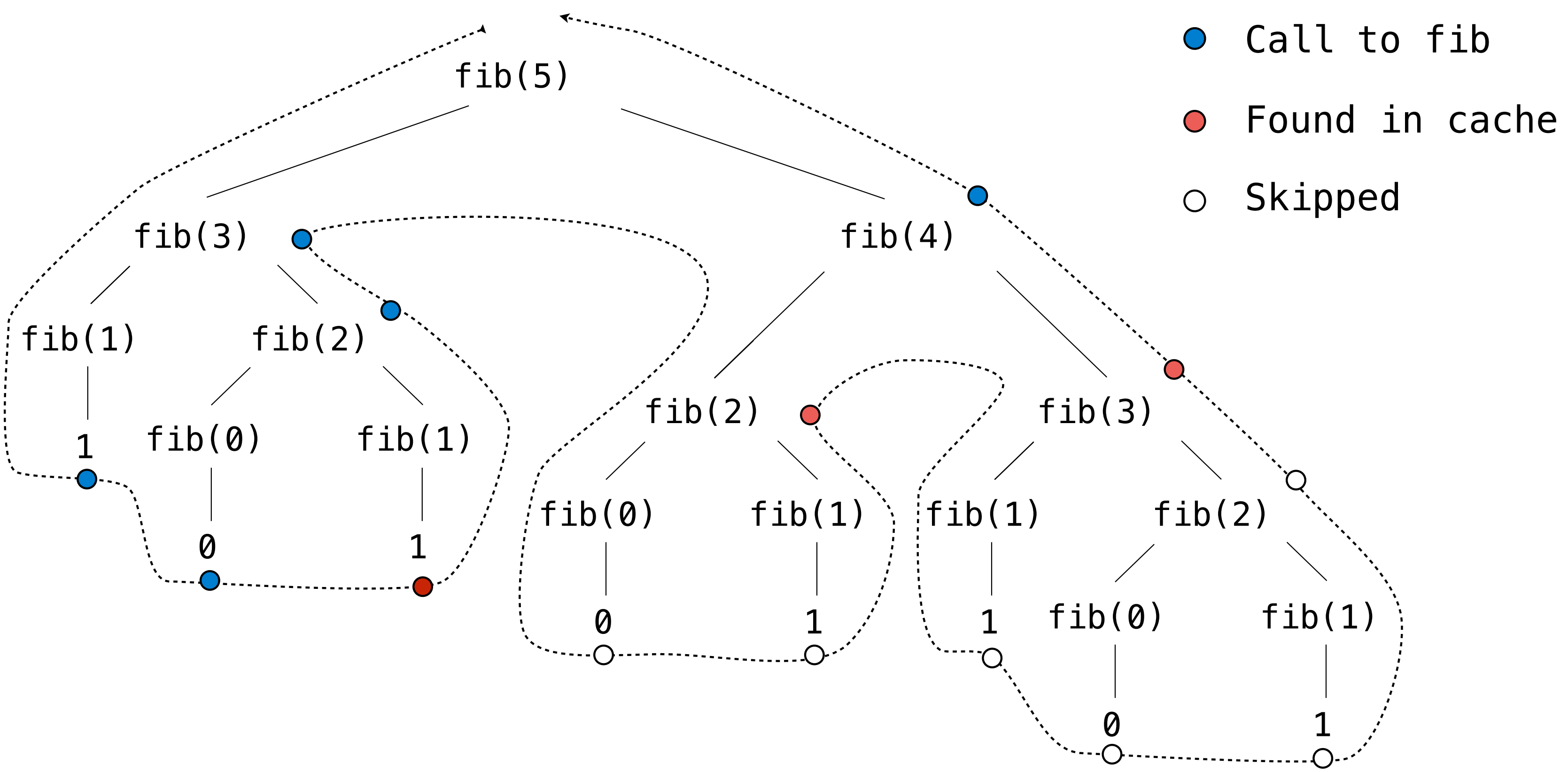

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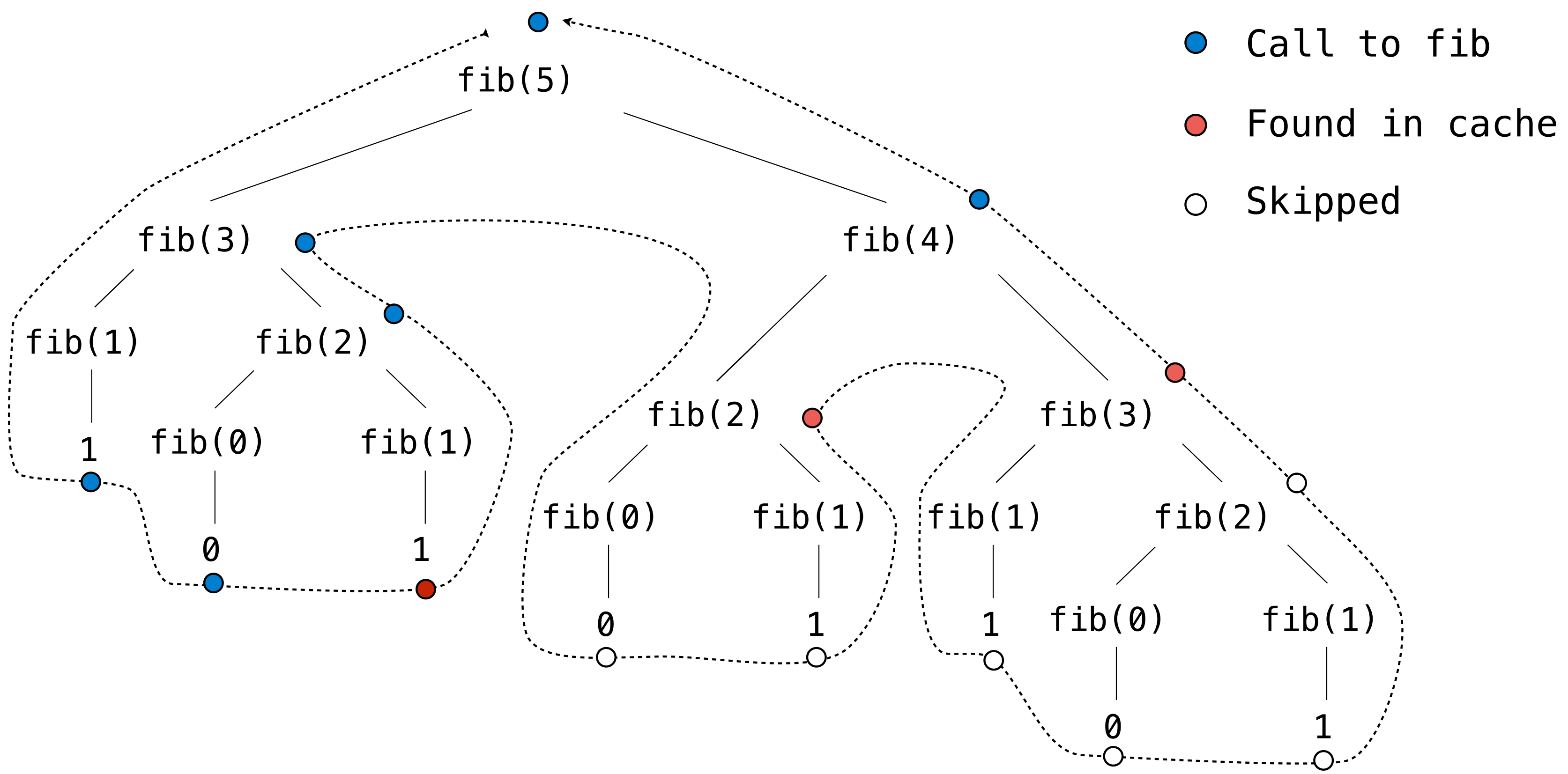
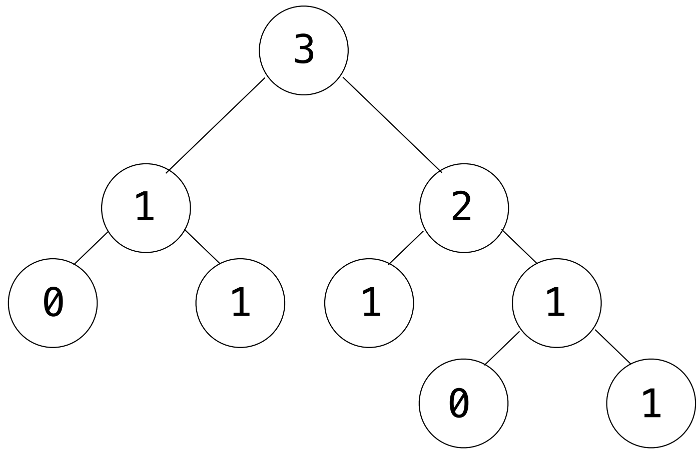


Tree Class

Tree Abstraction (Review)



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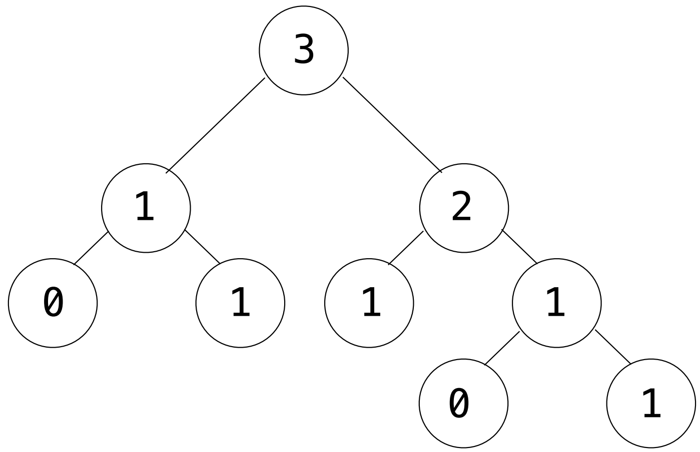

Recursive description (wooden trees):

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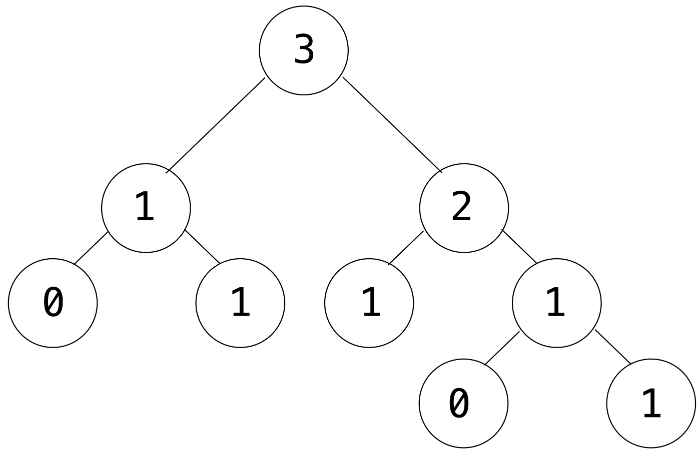

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A **tree** has a **root label** and a list of **branches**

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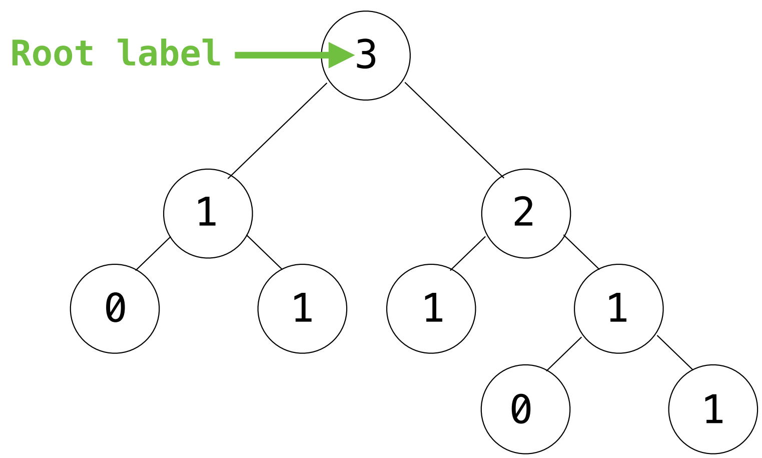

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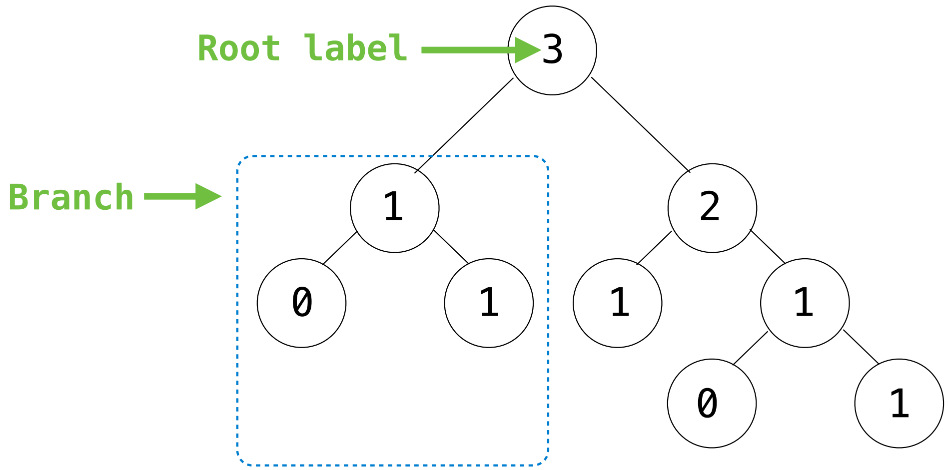

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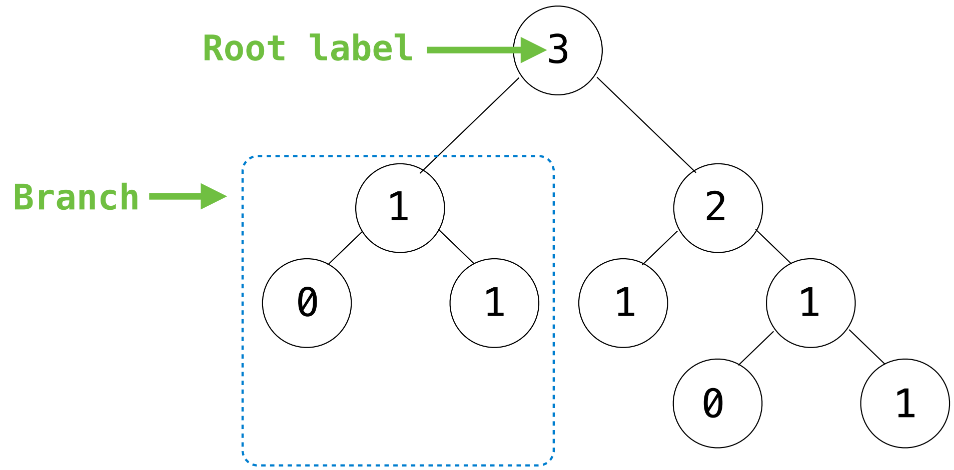

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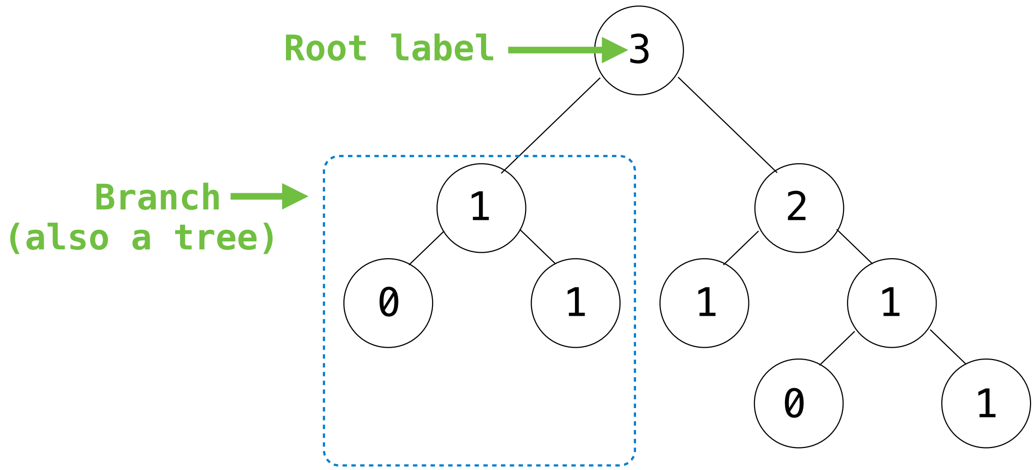

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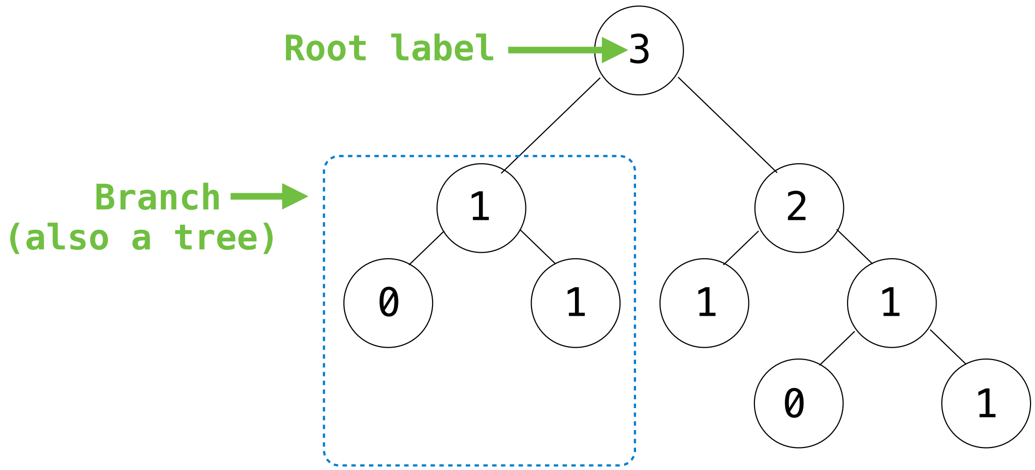

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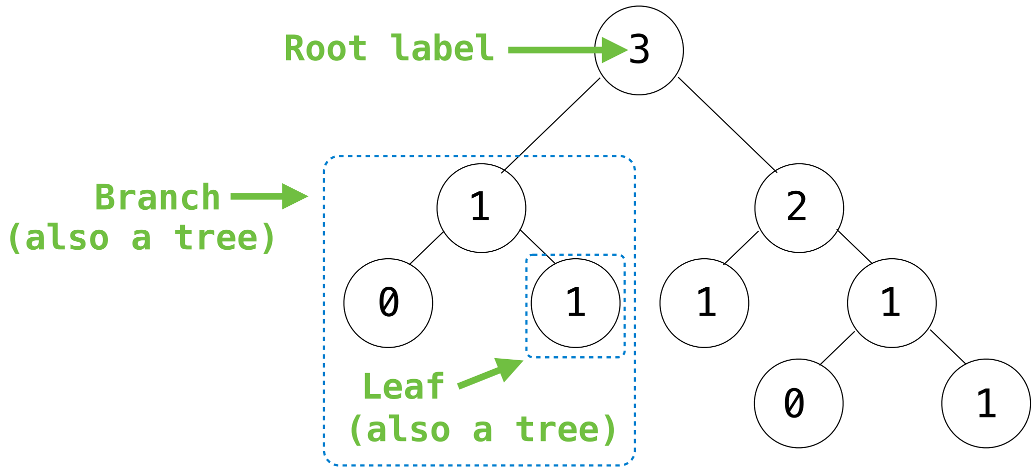

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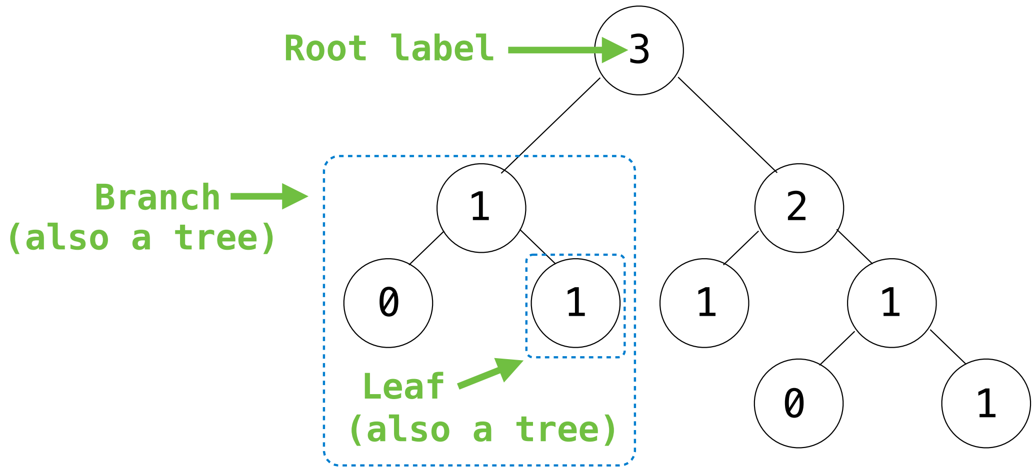

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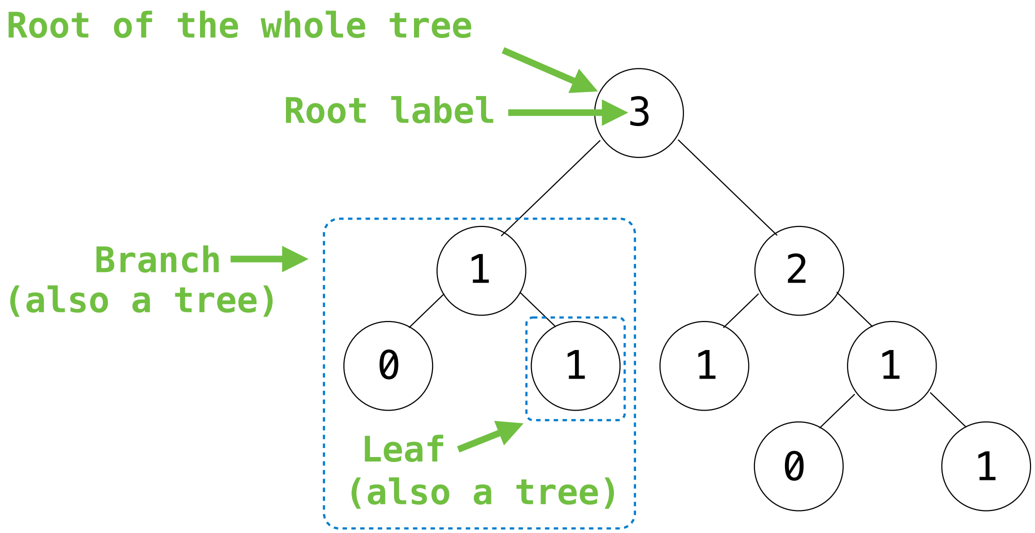

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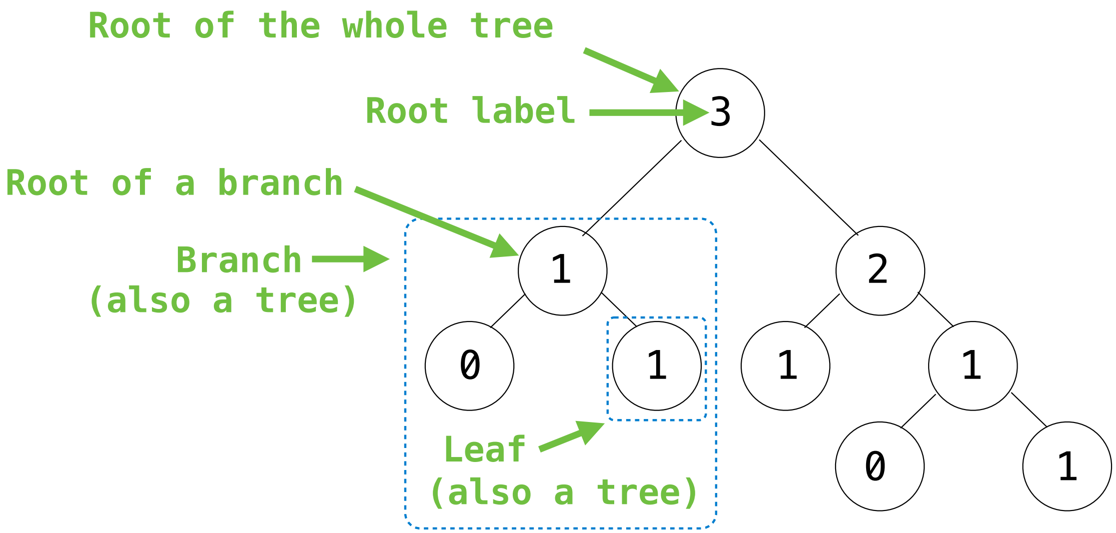

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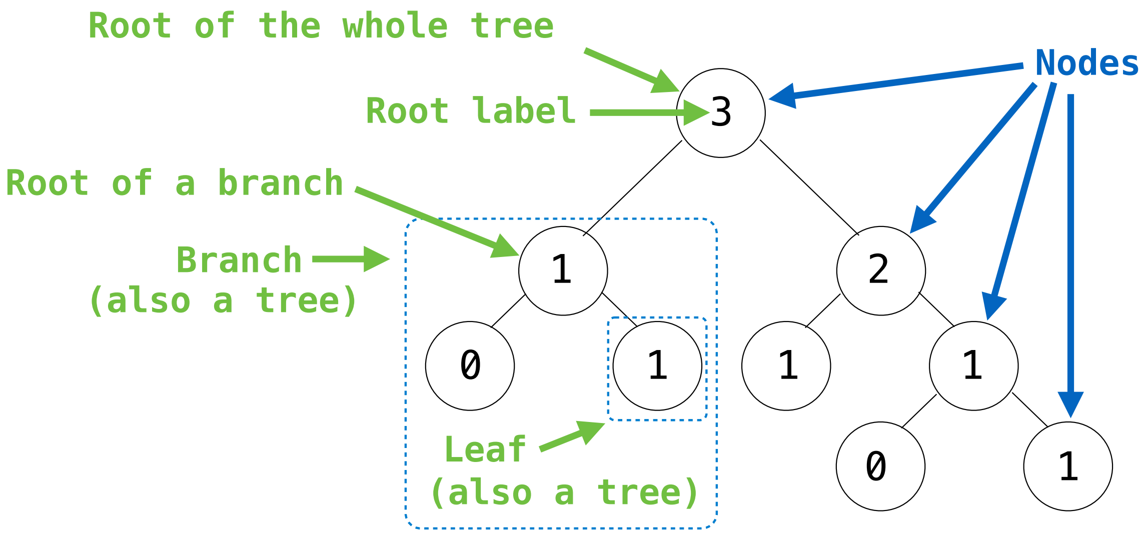

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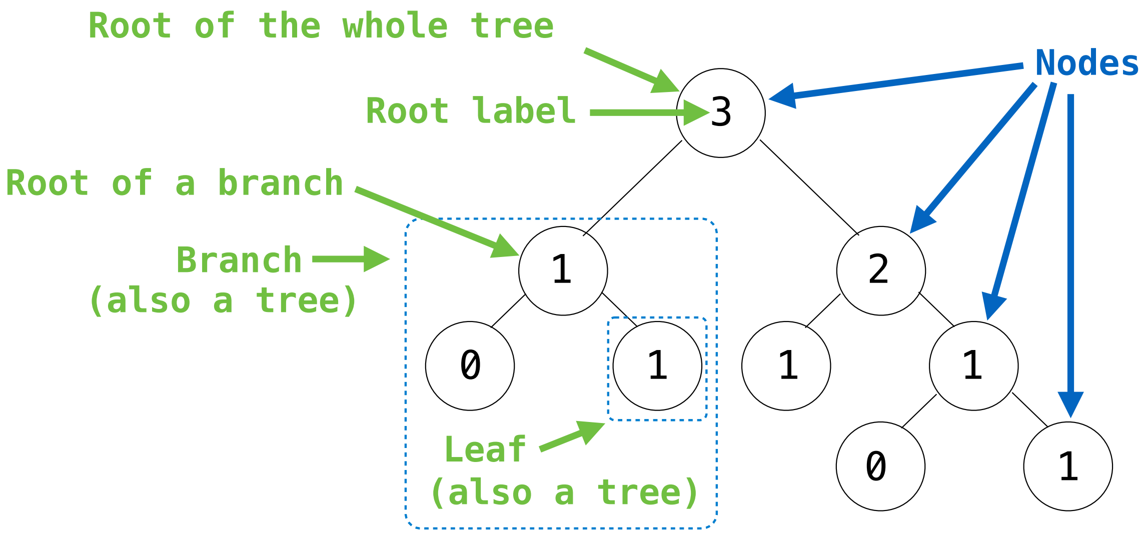

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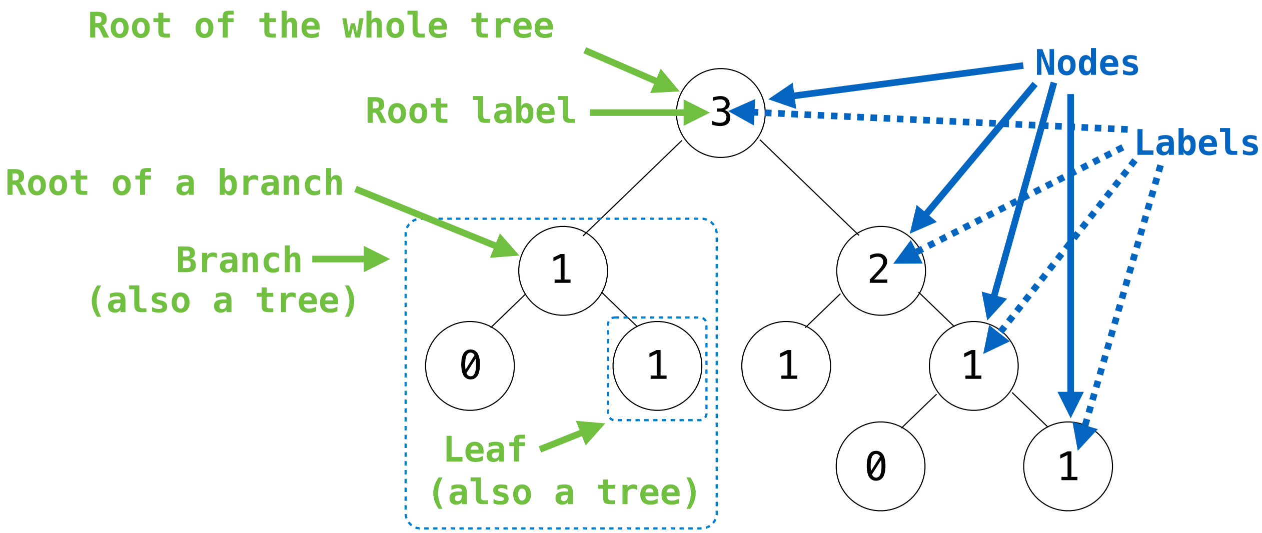

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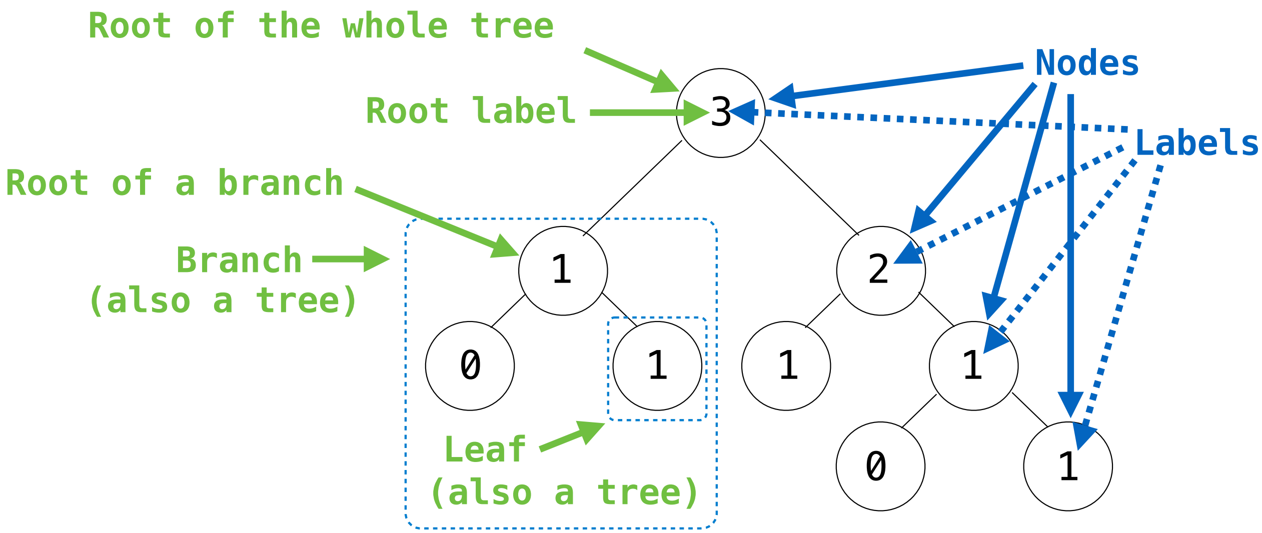

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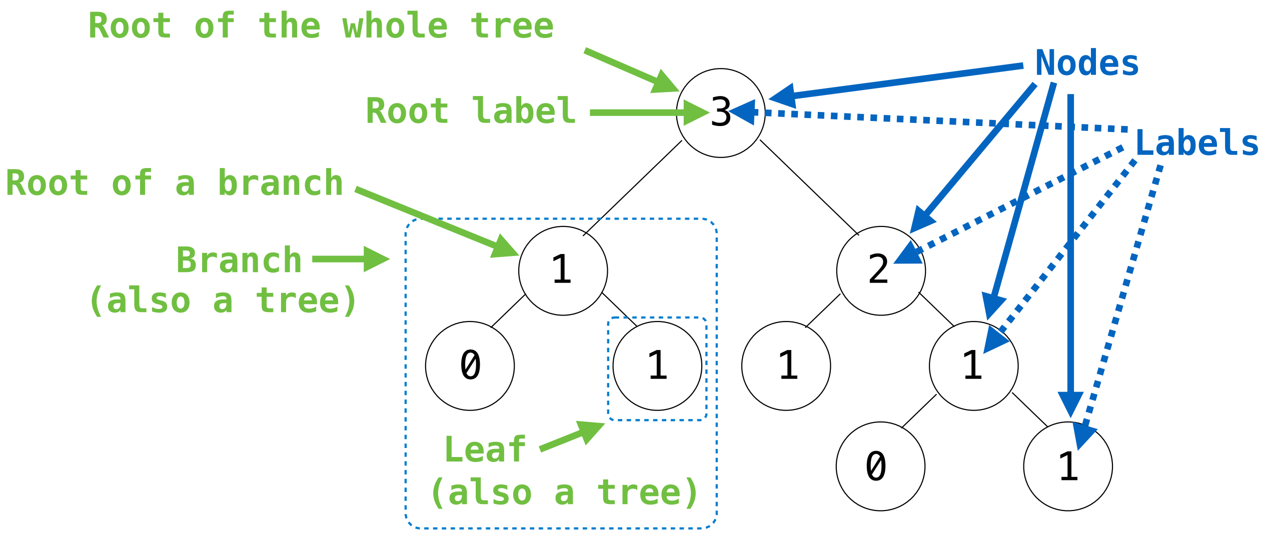

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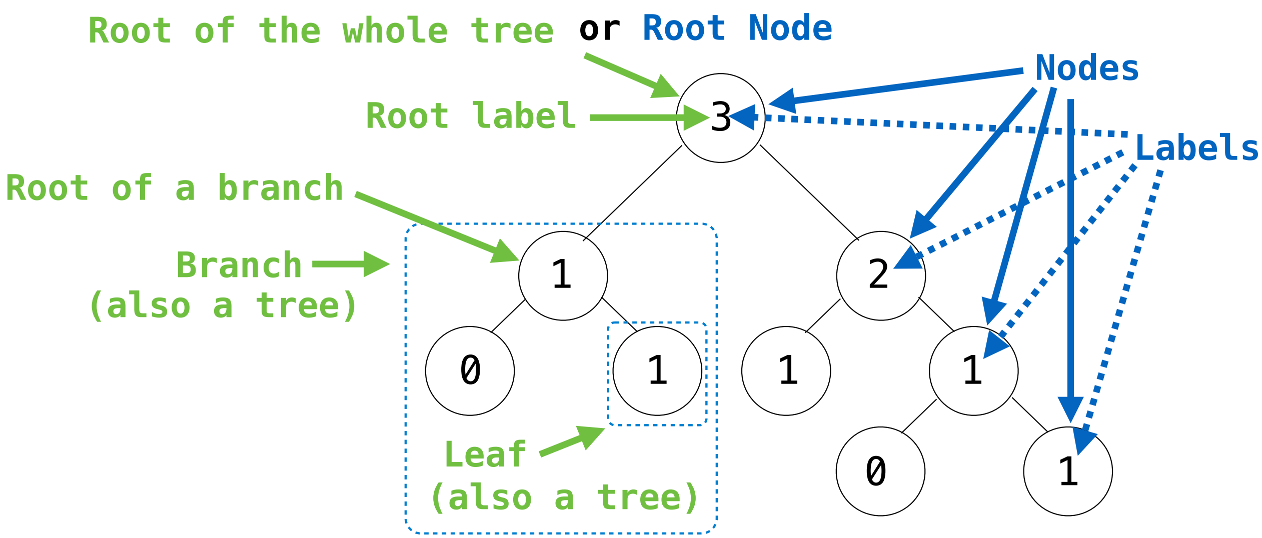

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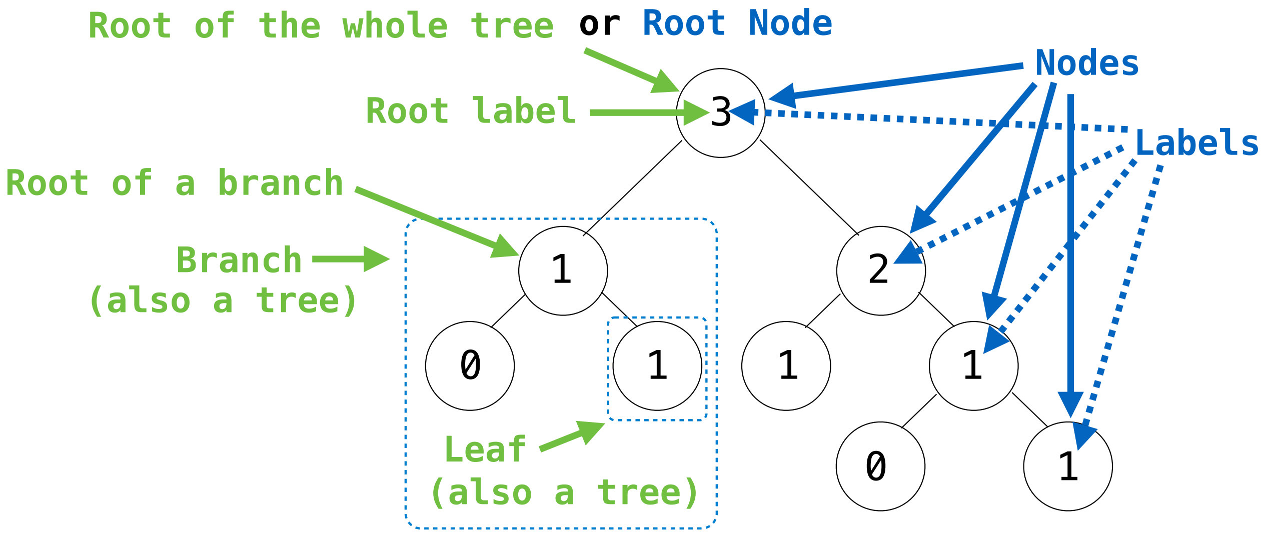

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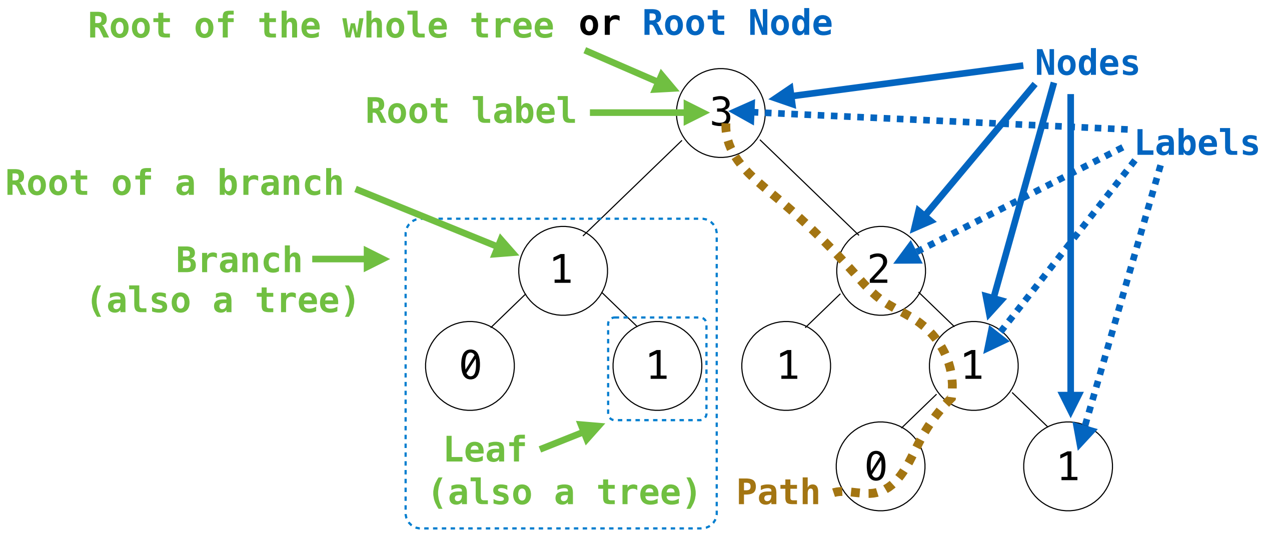

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    for branch in branches:
        assert is_tree(branch)
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def label(tree):
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```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label + right.label
        return Tree(fib_n, [left, right])
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        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
```

```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label + right.label
        return Tree(fib_n, [left, right])
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = label(left) + label(right)
        return tree(fib_n, [left, right])
```

Tree Class

A Tree has a label and a list of branches; each branch is a Tree

```python
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
```

```python
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
def label(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
```

```python
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label + right.label
        return Tree(fib_n, [left, right])
```

```python
def fib_tree(n):
    if n == 0 or n == 1:
        return tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = label(left) + label(right)
        return tree(fib_n, [left, right])
```

(Demo)